Journal of Electronics and Informatics (ISSN: 2582-3825)
www.irojournals.com/iroei/

Radial Differentiation of Pump and Signal Intensities in Trapezoidal index EDFA for LP_{11} mode in Kerr nonlinear state

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Abstract

An Erbium-Doped Fiber Amplifier (EDFA) is an in-line component in modern all –optical telecommunication infrastructure. Different parametric characteristics of an EDFA express the suitability and excellency of performance in its real field application. Intensities of pump and signal vary with distance from the core-axis along the radius of the fiber which is one of the significant characteristics of an EDFA. Change of behavior of pump and signal intensities along the radius of the fiber in an erbium-doped dual-mode trapezoidal index fiber made amplifier due to Kerr nonlinearity phenomenon originating from launching and transmission of intense light from LASER beam inside the amplifier for the LP_{11} mode has been exercised in this case. In the present case, some trapezoidal-index fibers of different normalised frequencies have been opted. This exercise is an implementation of the reliable and easy mathematical instrument, the Chebyshev technique. Results derived in this exercise exhibit a fantastic similarity with those derived by the rigorous finite element method. This study with implementation of such a reliable and easy technique may help the interested optical engineers.

Keywords: EDFA, trapezoidal-index fiber, dual-mode, intensity of LP_{11}mode, Kerr nonlinear state, Chebyshev technique

1. Introduction

Since the invention of EDFA application of the complicated opto-electronic device, repeaters have been limited. In modern all-optical telecommunication systems and networks, the powerful device EDFA is being used to revive power for

DOI: https://doi.org/10.36548/jei.2022.3.006
Received: 06.08.2022, received in revised form: 21 .08.2022, accepted: 30.08.2022, published: 09.09.2022

source and detector in all-optical communication systems. EDFAs are inserted at specific gaps along the fiber-path in long distance telecommunication channel to revive the depreciated transmitted optical signal in linearity. Rare earth dopants dispense the gain in EDFA and it facilitates high gain for a broad wavelength span of the optical signal. For this spaciality fiber amplifier is highly fitting for modern photonic telecommunication network. High gain of 40 to 45 dB with low noise between 3 and 4 dB have been demonstrated [1,2] with 50 to 100 mW range of optical pump power. Such devices are made for lasing over the higher wavelength span (1500 to 1600nm) of interest in fiber-optic telecommunications. Realistically pump bands operate at wavelengths of 532nm, 670 nm, 807 nm, 980 nm and 1480 nm [3,4]. However, the greatest useful pump wavelengths are 980 nm and 1480 nm. In the wavelength division multiplexing (WDM) systems, the signal of wavelength span between 1525nm and 1600nm requires to be amplified and it is amplified by erbium-doped fiber amplifier (EDFA). Improved efficiency is obtained by applying readily available pump sources at wavelength of 1480 nm.

Differentiation of power of signal and pump beams with separation distance from the core-axis in doped step-index fibers made EDFA for fundamental mode has been estimated accurately [5]. Intensity change for LP_{11}mode in graded-index (GI) fiber made EDFA is also an interesting publication in the literature [6]. The simple series form for the field of LP_{11} mode in a GI fiber has motivated the researchers in the concerned area [7] and its veracity has been established in the estimation of various significant optical measurements [7,8].

Refractive index profiles of doped optical fibers are changed in the Kerr nonlinear state of the fibers. The transmission parameters of these fibers show different values in the nonlinear state from those in the linear state [9-17]. Distribution of the intensities of the signal and pump along the radius of the fiber in Kerr nonlinear condition for the LP_{11} mode in an erbium-doped fiber amplifier (EDFA) consisting of dual-mode step index (SI) fiber is not yet reported in the literature. Using a user-friendly technique of Chebyshev formalism, the intensity distributions of signal and pump has been predicted along the radius of the fiber in the Kerr nonlinear condition for the LP_{11} mode in an EDFA made up of a dual-mode SI fiber. In the Kerr nonlinear condition radial distribution of intensities for signals having wavelengths of 1550 and 1580 nm and pump of wavelength1480 nm have been estimated in our study. In case of WDM technology in the long haul all-optical communication system this study is extremely important. Till date this type of study has not been reported in the literature. Thus, this study involves uniqueness and it will be helpful for the optical technologists.

2. Theory

For a GI optical fiber having core-axis index of refraction n_1 and clad index of refraction (r.i.) n_2 the index variation is mathematically represented as,

$$n^2(R) = n_1^2\big(1 - 2\delta f(R)\big), \qquad 0 < R \le 1$$
$$= n_2^2, \qquad > 1 \tag{1}$$

Here, $\delta = (n_1^2 - n_2^2)/2n_1^2$ and R = r/a ('a' is the core radius and 'r' is the distance from the core-axis measured radially). The form of the RI contour of the fiber is defined by f (R).

For trapezoidal-index fiber [18]

$$f(R) = 0, \qquad 0 < R \le S$$
$$f(R) = \frac{R-S}{1-S}, \qquad S < R \le 1 \tag{2}$$

Here, S is termed as trapezoidal fibre's aspect ratio.

The RI profile *n(R)* of an optical fiber in Kerr nonlinear condition is given by [17],

$$\mathrm{n}^2(\mathrm{R}) = \mathrm{n}_\mathrm{L}^2(\mathrm{R}) + \frac{\mathrm{n}_2^2\mathrm{n}_\mathrm{NL}(\mathrm{R})}{\eta_0}\psi^2(\mathrm{R}) \tag{3}$$

Here,$\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$, ε_0 andμ_0 are the permittivity and permeability of free space; $\mathrm{n_L}(\mathrm{R})$ is the RI profile in disappearance of nonlinearity, $\mathrm{n_{NL}}(\mathrm{R})$ is the span of nonlinear Kerr parameter and $\psi(\mathrm{R})$ is the field of the LP_{11} mode. The following scalar equation expresses the LP_{11} modal field $\psi(\mathrm{R})$ for a Kerr class nonlinear fiber [17]

$$\frac{\mathrm{d}^2\psi(\mathrm{R})}{\mathrm{dR}^2} + \frac{1}{\mathrm{R}}\frac{\mathrm{d}\psi(\mathrm{R})}{\mathrm{dR}} + [\mathrm{V}^2\{1 - \mathrm{f}(\mathrm{R})\} - \mathrm{W}^2]\psi(\mathrm{R}) - \frac{\psi(\mathrm{R})}{R^2} + \mathrm{V}^2\mathrm{g}\psi^3(\mathrm{R}) = 0 \tag{4}$$

Here, *g is* represented mathematically as

$$\mathrm{g} = \frac{\mathrm{n_2 n_{NL} P}}{\pi \mathrm{a}^2(\mathrm{n}_1^2 - \mathrm{n}_2^2)}$$

V is the normalized frequency and W is the clad decay parameter in Eq.(4).

The wave function continues at the core-clad interface (r = a) when,

$$\left[\frac{1}{R}\frac{d\psi}{dR}\right]_{R=1} = -\left[1+\frac{WK_0(W)}{K_1(W)}\right] \tag{5}$$

Here, $K_0(W)$ and $K_1(W)$ are the modified Bessel functions of W [19-21].

Following Chebyshev technique [7], the field $\psi(R)$ for the LP_{11} mode in GI fiber can be expressed in approximation as,

$$\begin{aligned}\psi(R) &= a_1R + a_3R^3 + a_5R^5, \qquad R \le 1\\ &= (a_1+a_3+a_5)\frac{K_1(WR)}{K_1(W)}, \qquad R > 1\end{aligned} \tag{6}$$

where a_1, a_3 *and* a_5 are constants.

In general, the Chebyshev values are given by [22]

$$R_m = \cos\left(\frac{2m-1}{2M-1}\frac{\pi}{2}\right) \quad m = 1, 2, \dots, (M-1) \tag{7}$$

Considering Eq. (6) and taking M =3 one can evaluate concerned Chebyshev values from Eq. (7) as

$$R_1 = 0.9511, \quad R_2 = 0.5878 \tag{8}$$

The following two equations by using these two important Chebyshev values along with Eqs. (6) and (4)

$$a_1\left[V^2\big((1-f(R_1)\big)-W^2+V^2g\psi^2(R_1)\right]+a_3\left[8+R_1^2\left\{V^2\big((1-f(R_1)\big)-W^2+V^2g\psi^2(R_1)\right\}\right]+a_5\left[24R_1^2+R_1^4\left\{V^2\big((1-f(R_1)\big)-W^2+V^2g\psi^2(R_1)\right\}\right]=0 \tag{9}$$

and

$$a_1\left[V^2\big((1-f(R_2)\big)-W^2+V^2g\psi^2(R_2)\right]+a_3\left[8+R_2^2\left\{V^2\big((1-f(R_2)\big)-W^2+V^2g\psi^2(R_2)\right\}\right]+a_5\left[24R_2^2+R_2^4\left\{V^2\big((1-f(R_2)\big)-W^2+V^2g\psi^2(R_2)\right\}\right]=0 \tag{10}$$

Plotting of $K_1(W)/K_0(W)$ versus $1/W$ for values of W in the range between 0.6 and 2.5 is adequately linear [20]. This allows us to create the following relationship in the stated range using the linear least square fitting approach [23]

$$\frac{K_1(W)}{K_0(W)} = \alpha + \frac{\beta}{W} \quad (11)$$

with values of α and β as 1.034623 and 0.3890323 respectively.

Combining Eqs. (6), (11), and (5), the following equation is derived,

$$a_1[2(\alpha W + \beta) + W^2] + a_3\,[4\,(\alpha W + \beta) + W^2] + a_5\,[6(\alpha W + \beta) + W^2] = 0 \quad (12)$$

For a nontrivial solution for the constants from the Eqs. (9), (10), and (12) following condition has to obey,

$$\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = 0 \quad (13)$$

where,

$$A_1 = V^2\{1 - f(R_1)\} - W^2 + V^2 g\psi^2(R_1)$$

$$A_2 = V^2\{1 - f(R_2)\} - W^2 + V^2 g\psi^2(R_2)$$

$$A_3 = 2(\alpha W + \beta) + W^2$$

$$B_1 = 8 + R_1^2[V^2\{1 - f(R_1)\} - W^2 + V^2 g\psi^2(R_1)]$$

$$B_2 = 8 + R_2^2[V^2\{1 - f(R_2)\} - W^2 + V^2 g\psi^2(R_2)]$$

$$B_3 = 4(\alpha W + \beta) + W^2$$

$$C_1 = 24R_1^2 + R_1^4[V^2\{1 - f(R_1)\} - W^2 + V^2 g\psi^2(R_1)]$$

$$C_2 = 24R_2^2 + R_2^4[V^2\{1 - f(R_2)\} - W^2 + V^2 g\psi^2(R_2)]$$

$$C_3 = 6(\alpha W + \beta) + W^2$$

Taking $g = 0$, clad decay parameter W can be evaluated for the linear condition for any optical fiber of a selected type with a particular V number from the solution of Eq. (13). Applying this value of W for the specific normalized frequency in any two of the Eqs. (9), (10), and (12), the values of a_3 and a_5 are derived in terms of a_1 for the specific V number in linear condition. Thus, from Eq.(6) field $\psi(R)$ for LP_{11} mode in that type of fiber in the linear condition is achieved. Then to derive W for a specific value of g (corresponding to the specific normalized frequency) for the selected fiber, the previously derived value of W in the linear condition is used in Eq. (13) and the iteration method is repeated again and again to find a concurrent value of W. Applying this concurrent value of W in any two of the Eqs. (9), (10,) and (12) concurrent values of $a_3$and a_5 are achieved relating to only a_1 in Kerr nonlinear condition with that specific value of g. Following the same procedure for all the selected fibers the modal fields ψ(R) of the LP_{11} mode in the linear and Kerr nonlinear conditions are predicted.

From Eq.(6) and replacing R by r/a, modal intensity in the core is obtained as,

$$\Psi^2(r) = a_1(r/a)^2 + 2a_1\, a_3(r/a\,)^4 + (\,a_3{}^2 + 2a_1a_5)(r/a\,)^6 + 2\; a_3a_5(r/a\,)^8 + a_5{}^2(r/a)^{\,10}, \quad r \le a \qquad (14)$$

and the modal intensity in the cladding,

$$\Psi^2(r) = (a_1 +\; a_3 + a_5)^2 \frac{K_1{}^2\left(\frac{Wr}{a}\right)}{K_1^2(w)} \qquad r > a \qquad (15)$$

3. Results and Discussion

The opted erbium-doped fiber amplifiers are fabricated with dual-mode trapezoidal-index fibers having the same core- radius 'a' of 3 μm and different V numbers commensurate with LP_{11} mode. In this study, signal wavelength of 1550 nm and pump wavelength of 1480 nm are used. The differentiation of intensity $\Psi^2(r)$for the excited LP_{11}mode for both the signal and pump with distance (r) measured radially from the core-axis in the aforesaid fibers both in nonlinear and linear states are estimated using Eqs. (15) and (16).

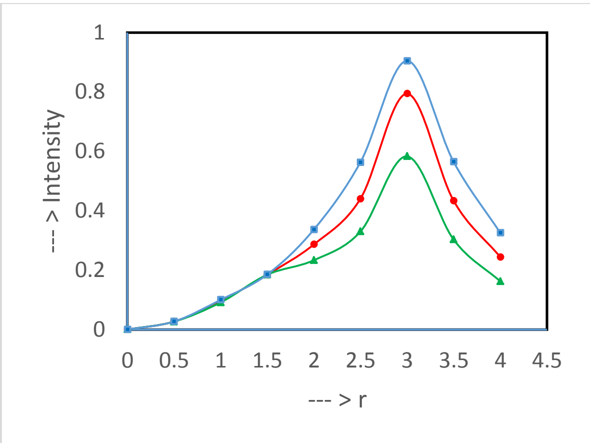

Fig. 1. Distribution of intensity' $\Psi^2(r)$ for pump at wavelength of 1480 nm for LP_{11} mode in an EDFA consisting of Trapezoidal-index fiber with S= 0.25,*V = 4.0*, a = 3μm,NA = 0.3139 for different nonlinearity parameters $n_{NL}P$ (Our results: ▲ for $n_{NL}P$ = +1.5X10^{-14}m^2 ,● for $n_{NL}P$ = 0 and ■ for $n_{NL}P$ = -1.5X10^{-14}m^2; —— simulated exact results)

Fig.1 depicts the differentiation of intensity for pump wavelength of 1480 nm in an EDFA consisting of Trapezoidal-index fiber with S= 0.25,V = *4.0* in linear and two different nonlinear conditions.

Fig.2 depicts the differentiation of intensity for that pump with S= 0.5,V = *3.25* in the same three states.

Fig.3 depicts the differentiation of intensity for signal wavelength of 1550 nm in trapezoidal-index fiber made EDFA with S= 0.25,V = 3.8183 in the same linear and nonlinear conditions.

In Fig.4, differentiation of intensity for signal wavelength of 1550 nm with S= 0.5,V = 3.1 has been exhibited.In every case solid lines ______ denote the accurate modal intensity$\Psi^2(r)$ derived by application of the finite element method [24].

It is observed that in all cases results from this Chebyshev technique match superbly with the accurate results. Results for $n_{NL}P = 0$ (linear condition) are also in conformity with the accessible accurate outcomes [6,24]. Solution of a (3x3) determinant by the iterative method of Chebyshev technique is involved in this prediction. The study is significant in connection with prudent identification of this type of EDFA to apply it in the area of optical telecommunication and sensors. Moreover, it generates enough flexibilty for application in different branches of all-optical technology and nonlinear photonics.

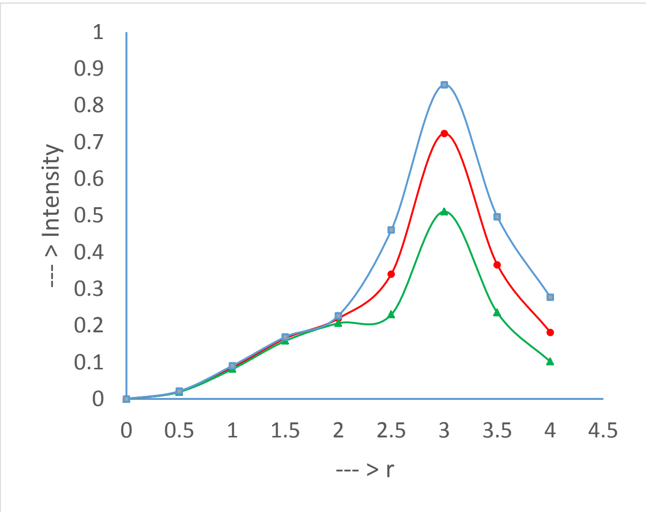

Fig.2. Distribution of intensity' $\Psi^2(r)$ for pump at wavelength of 1480 nm for LP_{11} mode in an EDFA consisting of Trapezoidal-index fiber with "S= 0.5,V = *3.25*, a = 3μm,NA = 0.2550 for different nonlinearity parameters $n_{NL}P$ (Our results: ▲ for $n_{NL}P$ = +1.5X10^{-14}m^2 ,● for $n_{NL}P$ = 0 and ■ for $n_{NL}P$ = -1.5X10^{-14}m^2; —— simulated exact results)"

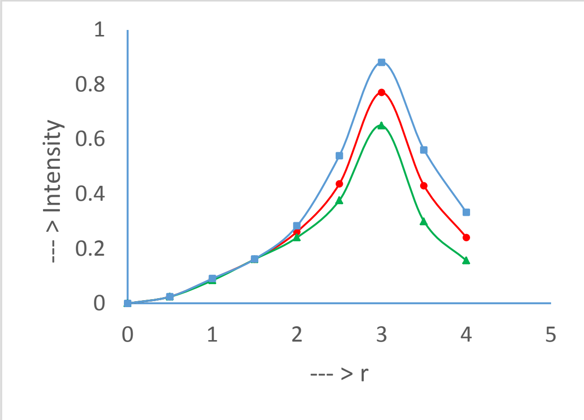

Fig.3. Distribution of intensity' $\Psi^2(r)$ for signal at wavelength of 1550 nm for LP_{11} mode in an EDFA consisting of Trapezoidal-index fiber with S= 0.25,V = *3.8183*, a = 3μm,NA = 0.3138 for different nonlinearity parameters $n_{NL}P$ (Our results: ▲ for $n_{NL}P$ = +1.5X10^{-14}m^2 ,● for $n_{NL}P$ = 0 and ■ for $n_{NL}P$ = -1.5X10^{-14}m^2; —— simulated exact results)

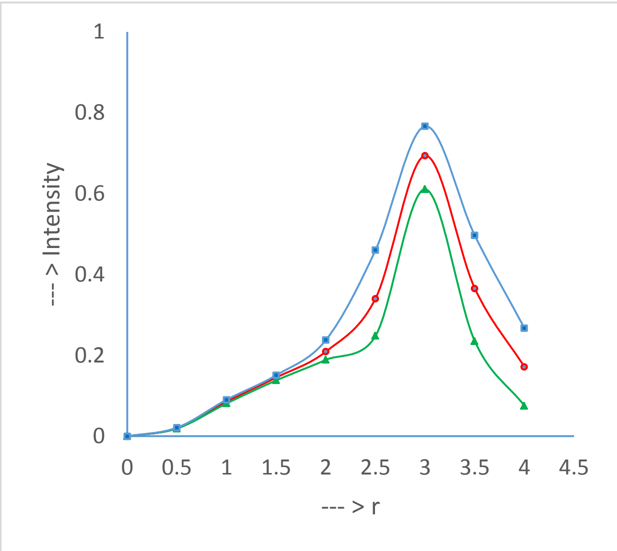

Fig.4. Distribution of intensity' $\Psi^2(r)$ for signal at wavelength of 1550 nm for LP_{11} mode in an EDFA consisting of Trapezoidal-index fiber with S= 0.5,V = *3.1*, a = 3μm,NA = 0.2548 for different nonlinearity parameters $n_{NL}P$ (Our results: ▲ for $n_{NL}P$ = +1.5X10^{-14}m^2 ,● for $n_{NL}P$ = 0 and ■ for $n_{NL}P$ = -1.5X10^{-14}m^2; —— simulated exact results)

4. Conclusion

Erbium-doped fiber amplifiers consisting of dual-mode trapezoidal-index fibers are taken with signal wavelengths at 1580 and 1550 nm and pump wavelength at 1480 nm for the study of the nonlinear effect on the radial variation of signal and pump intensity for the LP_{11} mode in the core and cladding. The evaluation is based on a unique formalism involving the iteration method. Positive nonlinearity decreases the signal and pump modal intensities, while negative nonlinearity enhances the modal intensities of signal and pump conforming the accuracy of the simple technique and the derived results. This study justifies to be highly convenient from the perspective of the application of EDFA in all-optical communication system and in the area of nonlinear photonic technology.

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