

Control Approaches through Time Weighted Error and Gain Margin tuning for Unstable Systems

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Abstract

The Proportional Integral Derivative (PID) control strategy is very simple, robust, and sufficient to tune the low-order plus time delay model. On the other hand, the empirical PID controller has structural limitations in controlling the unstable processes. The closed-loop dynamics and structural mismatch can be analyzed intuitively to understand why the PID controller is not suitable to control the unstable processes effectively. Also, the conventional PID controller with minor structural variations designed to guarantee Optimal Gain Margin (OGM) tuning and Integral of the Time-weighted Absolute Error (ITAE) tuning, handles better than the performance offered by a conventional PID controller for an unstable process.

Keywords: PID, Error Tuning, Unstable SOPTD, Optimal Gain margin tuning

1. Introduction

The characteristic gains of the Proportional Integral Derivative (PID) controller are placed with extensive consideration of the process dynamics. In the upshot, agreeable control performances cannot be effectuated. Imperfect tuning parameters would result in very stagnant or unstable responses to the process model.

Exhaustive random probing of controller gains is predominantly adopted to find the tuning parameters of a PID controller by analyzing the forceful behavior of the contained process output. It is essential to have knowledge of the tuning gains and their effects on the process output's characteristics and effects for fortunate trial-and-error tuning. The PID controller consistently records the following dynamic modes concerning the tuning parameters for the adjustment in step setpoint.

- If the proportional constant or gain, k_c is designed to be huge, then the process output prints a sizable oscillation.
- If the proportional constant or gain, k_c is designed to be diminutive, then the process output appears like the response of overdamped systems.
- If the integral time t_i is meager (that is, the integral action is too sturdy), the process output fluctuates and the process output prevails above the setpoint for a longer duration than under the setpoint.
- If the integral time t_i is immense, then the process output palpitates to make the process output stays under the setpoint beyond a time it prevails above the setpoint.
- If the derivative time is extreme, then the process registers oscillation from the outset to the steady-state.

The user can choose the gains of the PID controller in a cut-and-try manner by modifying the tuning parameters in the direction of minimizing the above-mentioned dynamic characteristics. The most important thing to be considered in the PID controller design for successful trial-and-error tuning is to sustain the proportional gain as high as possible. The closed-loop dynamics may become slower and slower during trial-and-error tuning if the focus is on zeroing the dynamic characteristic without attempting to sustain a large proportional gain. In this research work, simple control methods are registered to evaluate the efficient controller gains for the process model, with the aid of error and optimal gain tuning for an unstable process plus time delay.

2. Related Work

The tuning of PID controllers was initiated and went through a storm of discussion after Zeigler et al. [1] had given a set of tuning procedures for the three different configurations of the PID controllers. PID controller tuning was handled with the perspective of Integral Absolute Time-weighted performance criteria by Lopez et al. [3] and proved to be promising in terms of time-domain specifications. Sung S. W. et al. discussed the countermeasures and limitations of PID controllers for an industrial process that considered the time delay of the process into consideration. Kwak et al. [7] promoted the tuning of controllers and its significance in the control of unstable processes. M.Ghazali [8] formulated

the tuning of controllers using an evolutionary algorithm considering an unstable process. F.Gazdos [9] established a new tool for modeling unstable systems that aided in the evolution of the modeling parameters of the process. W.K.Ho et al. [10] instigated the optimal Gain margin-planted tuning of PID Controller for systems with and without time delay.

3. Proposed Work

3.1 Experimentation Process

The process considered for the test is as shown in equation (1) and is derived from a blender mechanism [1] and the transfer function is found to be an Unstable Second-Order Process with Time Delay (SOPTD).

$$G(S) = e^{-0.2s} * \frac{1}{(3s^2 + 2s - 1)}$$

The process has one pole located on the right half of the s-plane, a negative phase margin and thus the process under investigation is unstable. It is also characterized by a time delay of 0.2 seconds, which added the complexity of constructing a controller for effective performance measures of the process.

3.2 Zeigler-Nichols Tuning Rule

The first attempt in controlling the process is undertaken with the Zeigler-Nichols Tuning (ZN) rule as stated in Table 1.

Table 1. Zeigler- Nichols Tuning

Controller	Tuning parameters		
	Proportional Gain, Kc	Integral Gain, Ki	Derivative gain, Kd
P	Ku/2.0	-	-
PI	Ku/2.2	Pu/1.2	-
PID	Ku/1.7	Pu/2.0	Pu/8.0

The Ku and Pu in the Table indicate ultimate gain and ultimate period of the process respectively. The tuning rule needs only the ultimate data of the process and is very easy to

implement through a continuous-cycling method. The PID controller configuration was designed using this tuning rule and the numerical gains of controller parameters were calculated to be $K_p=5.097$; $k_i=4.634$; $k_d=5.997$.

3.3 Integral of the Time-Weighted Absolute value of Error Tuning Rule (ITAE)

The process taken under study is having a pole on the right half of the s-plane, which makes the system exhibit unstable characteristics and indicates that the process is underdamped. Hence the contemporary PID tuning methods like the Zeigler Nichols Tuning rule is not sufficient to assess the control performances associated with the process. Hence the search for alternate tuning methods pointed to the Integral of the Time-Weighted Absolute value of the Error Tuning Rule (ITAE) proposed by [6]. This tuning rule is stationed on the damping factor values and the proportion of Time delay to the time constant values of the unstable second-order transfer function of the process. The tuning gains of each PID parameter are formulated as in equation (2) below.

$$\begin{aligned}
 kk_c &= -0.04 + \left[0.333 + 0.949 \left(\frac{\theta}{\tau} \right)^{-0.983} \right] \zeta, \quad \zeta \leq 0.9 \\
 kk_c &= -0.544 + 0.308 \left(\frac{\theta}{\tau} \right) + 1.408 \left(\frac{\theta}{\tau} \right)^{-0.832} \zeta, \quad \zeta > 0.9 \\
 \frac{\tau_i}{\tau} &= \left[2.055 + 0.072 \left(\frac{\theta}{\tau} \right) \right] \zeta, \quad \frac{\theta}{\tau} \leq 1.0 \\
 \frac{\tau_i}{\tau} &= \left\{ 1.768 + 0.329 \left(\frac{\theta}{\tau} \right) \right\} \zeta, \quad \frac{\theta}{\tau} > 1.0 \\
 \frac{\tau}{\tau_d} &= \left\{ 1.0 - \exp \left[- \frac{(\theta/\tau)^{1.060} \zeta}{0.870} \right] \right\} \left[0.55 + 1.683 \left(\frac{\theta}{\tau} \right)^{-1.090} \right]
 \end{aligned}$$

The controller parameters were calculated to be $K_p=4.723$; $\tau_i=2.062$; $\tau_d=1.459$. The controller structure with the designed values is implemented and evaluated as in equation (3).

$$\begin{aligned}
 G_c(s) &= 4.723 + \frac{2.29}{s} + 6.890s \\
 G(s) &= e^{-0.2s} * \frac{6.89s^2 + 4.723s + 2.29}{3s^3 + 2s^2 - s}
 \end{aligned}$$

3.4 Optimal Gain Margin Tuning Rule (OGM)

The control of the unstable process is cumbersome for a conventional PID controller because of its structural limitations. Hence a restructured PID controller with a PD controller

in its internal feedback is recommended by [7], which utilizes the optimal data sets fitted to the model and the parameters are found.

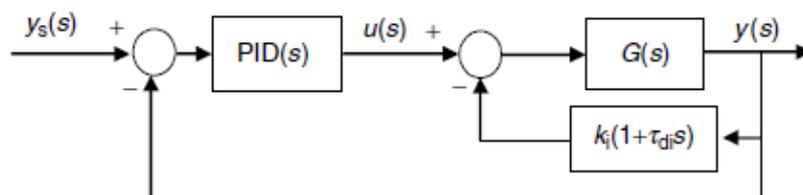


Figure. 1 Block Diagram of OGM with PD controller in internal feedback.

Figure 1 shows Restructured PID controller with a Proportional + Derivative (PD) controller on its internal feedback. The tuning rules are as tabulated in Table.2.

Table 2. The Optimal Gain Margin Tuning

Controller	Tuning parameters
k_i	τ_{di}/τ
P	$k_i = \frac{1}{\sqrt{ G_m(i\omega_u) G_m(0) }}$
PD	$k_i = \frac{1}{\sqrt{ G_m(i\omega_u)(1 + i\tau_{di}\omega_u) G_m(0) }}$ $\frac{\tau_{di}}{\tau} = X_1 + X_2 \left(\frac{\theta}{\tau}\right) + X_3 \left(\frac{\theta}{\tau}\right)^2$ $X_1 = -0.003 + 0.6482 \left(\frac{\tau_s}{\tau}\right) - 2.2841 \left(\frac{\tau_s}{\tau}\right)^2 + 2.6221 \left(\frac{\tau_s}{\tau}\right)^3 - 0.9611 \left(\frac{\tau_s}{\tau}\right)^4$ $X_2 = 0.2446 - 1.0410 \left(\frac{\tau_s}{\tau}\right) + 13.6723 \left(\frac{\tau_s}{\tau}\right)^2 - 16.7622 \left(\frac{\tau_s}{\tau}\right)^3 + 5.1471 \left(\frac{\tau_s}{\tau}\right)^4$ $X_3 = 0.1685 + 0.8289 \left(\frac{\tau_s}{\tau}\right) - 9.3630 \left(\frac{\tau_s}{\tau}\right)^2 + 2.9855 \left(\frac{\tau_s}{\tau}\right)^3 + 7.3803 \left(\frac{\tau_s}{\tau}\right)^4$

The controller parameters applied to the Optimal Gain margin rule and the structure of the internal feedback PD controller are designed to be in equation (4). The internal feedback controller is implemented to minimize the controller efforts and to achieve improved controller performances given stability with dynamic characteristics introduced due to the presence of time delay. The Optimal gain margin tuning method is extensively applied to the unstable process when the time delay associated with the process is between the value of zero and the value obtained by the difference between the time constant and sampling time.

$$K_p = 3.193G_c(S) = 3.193G(S) = e^{-0.2s} * \frac{3.193}{3s^2 + 2s - 1}$$

$$K_p = 7.162$$

$$K_d = 2.1486$$

$$G_c(S) = 2.1486s + 7.162$$

$$G(S) = e^{-0.2s} * \frac{2.1486s+7.162}{3s^2+2s-1}$$

4. Results and Discussion

The stated PID tuning methods are applied for the Unstable Second-Order Process with Time Delay (SOPTD). The step response of the ZN, ITAE, and OGM approach of tuning the PID controller is shown in Figure 2,3,4 respectively. These records that the optimal gain margin (OGM) method is outperforming in terms of magnitude when compared with the Integral-time Absolute error (ITAE) tuning and Zeigler – Nichols (ZN) tuning but undergoes a disturbing oscillation though the magnitude of the oscillation is comparatively smaller.

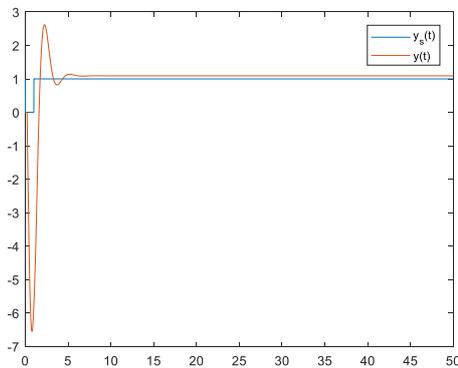


Figure 2. Step response for ZN tuning

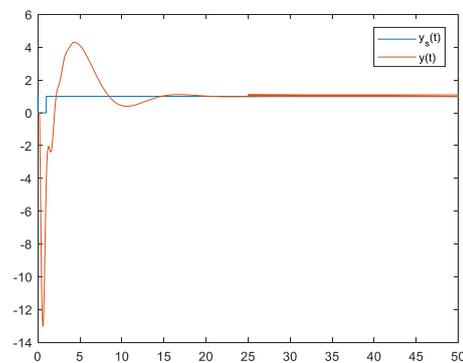


Figure 3. Step response for ITAE tuning

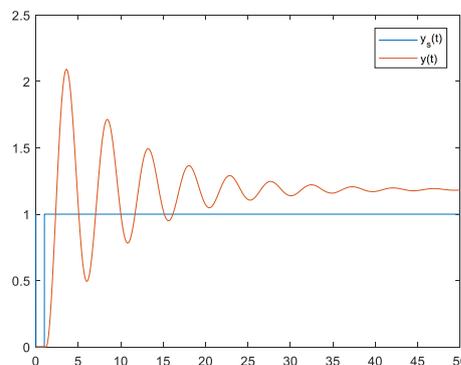


Figure 4. Step response for OGM tuning

The control action exhibited by the three controller configurations is as in Figures 5,6 and 7 respectively. These figures show that the effort made by the ZN and ITAE controller is overwhelmingly large in contrast to the controller effort offered by the optimal gain margin tuning (OGM) method. The magnitude of the controller action being very less for the OGM tuning recommends the method for the effective control of the unstable process.

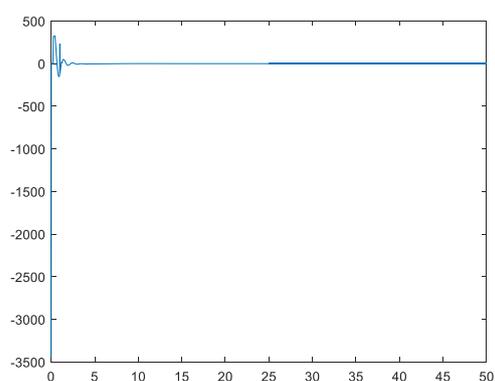


Figure 5. Controller Action for ZN tuning

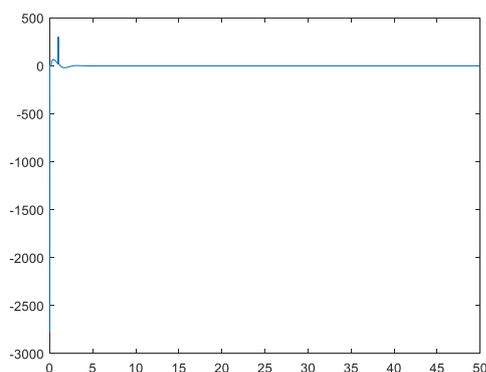


Figure 6. Controller Action for ITAE tuning

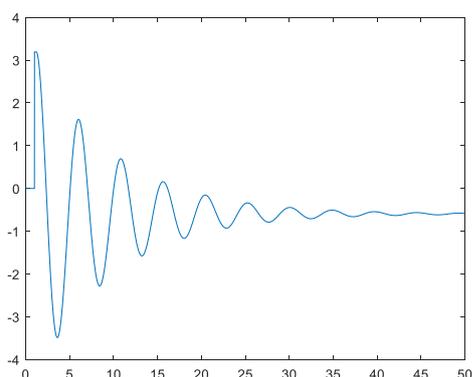


Figure 7. Controller Action for OGM tuning

Table 3. Frequency Response Specifications

	Process	ZN	ITALY	OGM
GM (db)	12	-333	-316(-19.5)	1.96
PM (deg)	-71.9	inf	inf	inf
PCF (rad/sec)	9.55	15.7	15.7(7.24)	9.55
GCF (rad/sec)	14.4	-	-	11.3

5. Conclusion

If the dynamic characteristics of the process are simple and a randomly tuned PID controller is sufficient to meet the control requirements, then the trial-and-error tuning is sufficient. For a more unstable process with time delay, the ZN tuning rule needs the conclusive frequency and gain data of the process and the ITAE-2 needs the time delay and the time constant details from the SOPTD model. Among the ZN and the ITAE, ITAE prints the best tuning result for the step setpoint change problem as it enables quick settling compared with the other two controller configurations. Yet considering the frequency response specifications such as Gain Margin (GM), Phase Margin (PM), Gain Crossover frequency (GCF), and Phase crossover frequency (PCF), it is obvious that the OGM tuning method shows the best frequency specifications responses like those of the ITAE, as tabulated in Table 3. If the process is unstable and underdamped, then the Optimal Gain Margin (OGM) tuning rule is recommended.

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