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Robustness Analysis of Quaternion Fractional Moment Descriptors for Geometric Pattern Recognition Under Photometric Noise

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Abstract

Quaternion Fractional Moment (QFM) descriptors have been widely used for geometric pattern recognition because they can represent multi-channel data and have some degree of invariance. However, they have not been systematically tested for their robustness. It has been particularly unclear to what degree they are robust when the geometric pattern is corrupted by noise influenced by lighting variations, which are common in imaging modalities. A robustness analysis framework for QFM descriptors has been developed in photometric noise-controlled environments. The approach proposes to include bounded spatiotemporal perturbations in the feature extraction process to obtain several realizations of the descriptor for a single geometric pattern. A stability measure indicative of the degree to which a particular feature tends to spread out from the mean of the descriptor, in the context of feature dispersion, ensures that robustness is measured directly in feature space. The classification accuracy is compared to a deterministic QFM and the stability behaviors of the two are linked to that of the QFM. Numerous experiments are conducted on Moroccan-Andalusian geometric patterns with p4m and p6m symmetries. It is supplemented with additive Gaussian noise of varying intensities for photometric stability. As inferred from the results, the proposed approach is able to improve stability in the descriptors with noise and ensure stable performance in recognition accuracy.

Keywords: Quaternion Fractional Moments, Geometric Pattern Recognition, Photometric Noise, Descriptor Robustness, Feature Stability Analysis, Symmetric Patterns, Cultural Heritage Imaging.

1. Introduction

Geometric Pattern Recognition focuses on the analysis of geometric structures defined by repetition, symmetry, and geometric organization in space. Like many other image analysis tasks, geometric pattern analysis has numerous applications, such as ornamental image analysis, texture recognition, and cultural heritage digitization. While object-related vision applications concentrate on well-delineated boundaries or meaningful geographic regions in images, geometric pattern analysis relies on well-delineated relationships among these geographic structures throughout the image. Nonetheless, in realistic imaging conditions, geometric patterns are not observed in perfectly illuminated conditions. This is due to photometric degradation that affects imaging on different photometric attributes such as

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luminosity. Feature-based recognition methods face issues, as even moderate noise could introduce further instability in the descriptor leading to reliability concerns in recognition. Quaternion-based moment descriptors have emerged as a great solution for representing geometric patterns in color images as they provide a unified algebraic framework to classify multi-channel patterns. The fractional-order extensions offer flexibility in further controlling spatial configuration for global representation and insensitivity to regional details. As a result, geometric pattern recognition and related applications have recently been focusing on Quaternion Fractional Moments (QFM). Existing evaluations of QFM descriptors, although demonstrating discriminative capability, have tended to focus on classification performances under a deterministic feature extraction pipeline. The descriptor-level measure of robustness to photometric noise is often simply inferred from a change in recognition performance. Thus, it is uncertain whether performance differences arise due to the inherent instability of the descriptors or due to the sensitivity of the downstream classifier. This research provides a counterargument to the above statement through a robustness-oriented analysis of the QFM descriptors with respect to photometric noise. Therefore, it is unclear whether performance differences result from the downstream classifier's sensitivity or from the descriptors' inherent instability. Through a robustness-oriented analysis of the QFM descriptors concerning photometric noise, this study offers a refutation of the aforementioned claim. Instead of treating feature extraction as a deterministic process, we allow for multiple realizations of the same feature for any given geometric pattern by introducing bounded spatiotemporal perturbations during descriptor computation. A feature's robustness in the feature space is directly quantified by the stability measurement, which assesses the degree of feature dispersal in a multi-iteration extraction cycle. A dataset of Moroccan-Andalusian geometric patterns with p4m and p6m crystallographic symmetries has been used to validate the analysis we proposed. Gaussian noise is added at various levels to simulate the photometric degradation that frequently occurs in real images. Comparative experiments with a deterministic QFM baseline are carried out to establish a connection between recognition performance and descriptor stability.

This paper makes the following three contributions:

- Proposes a framework for robustness analysis of QFM descriptors against photometric noise.
- A feature-space stability metric that measures the dispersion of descriptors.
- An empirical evaluation that demonstrates higher stability of the descriptors while preserving recognition performance.

This study emphasizes the significance of robustness-oriented evaluation in geometric pattern recognition and offers a clearer understanding of the behavior of QFM descriptors under non-ideal imaging conditions by clearly separating robustness analysis from classification accuracy.

2. Literature Review

The analysis of geometric motifs has been developed through multiple independent research trajectories rather than a single unified theoretical framework. The field has emerged from the convergence of mathematical symmetry theory, image analysis methodologies, and computer vision task formulations. Within the mathematical domain, foundational studies on planar tilings and symmetry groups established formal classification systems that remain central to contemporary analysis, particularly the categorization of wallpaper groups and the formal definition of translational, rotational, and reflection symmetries (Grünbaum & Shephard et al, Kaplan et al,). These formalisms provide a standardized descriptive framework that has been directly adopted by subsequent computational approaches.

Building on this foundation, extensive research has examined Islamic and Moroccan Andalusian geometric patterns as structured, rule-based systems amenable to analytical modeling and algorithmic reproduction. Early investigations focused on formal generative constraints governing arabesque constructions and symmetry-preserving design rules that ensure global structural consistency (Abas et al., Hankin & Hankin,). More recent studies have emphasized procedural construction methodologies, including the Tastir method, which models star polygons, rosettes, and self-similar tilings as recursive compositional systems governed by iterative geometric transformations. Complementary work has formalized artisanal construction practices as reproducible procedural workflows, demonstrating how Moroccan geometric patterns achieve crystallographic regularity while maintaining high ornamental density (Aboufadil et al., Castera, Thalal et al.,).

When the analytical focus shifts from rule-based construction to image-based observation, additional constraints related to data acquisition and representation become dominant. Images of identical geometric patterns may exhibit substantial variability due to changes in illumination, viewpoint, sensor noise, blur, surface degradation, and partial occlusion. Consequently, the problem extends beyond correct class assignment to the evaluation of whether a given representation yields invariant and repeatable structural descriptors under typical acquisition perturbations.

Early computer vision approaches employed handcrafted feature descriptors, particularly orthogonal moment-based representations such as Zernike moments. These descriptors encode spatial structure into compact feature vectors while providing rotation invariance, which is essential for pattern classes characterized by symmetry and repetition (Canterakis et al, Khotanzad & Hong et al,). Although Zernike moments have demonstrated discriminative effectiveness for Islamic geometric patterns under controlled imaging conditions (Ahadian & Bastanfard et al.,), higher-order moments are prone to numerical instability and exhibit performance degradation under noise, illumination variation, rotation, occlusion, and partial structural erosion.

Quaternion-based image representations have been increasingly adopted to address these limitations, particularly in scenarios where color information contributes significantly to structural discrimination. By representing multichannel color information within a unified algebraic space, quaternion-based methods preserve cross-channel correlations that are critical for accurate representation of ornamental structure, texture continuity, and edge integrity under uneven illumination and channel-specific noise. Survey studies in quaternion color image processing consistently report improved invariance properties and enhanced discriminative performance in multichannel pattern analysis tasks (Huang et al.,).

Fractional-order extensions of quaternion moment descriptors have been introduced to provide tunable control over the spatial-frequency characteristics of the representation. Quaternion Fractional Moments (QFMs) allow adjustable emphasis between global structural information and localized detail, enabling improved robustness to noise and geometric distortions through appropriate selection of fractional parameters (He et al.,; Yamni et al.,). However, existing evaluations predominantly assess performance through classification accuracy within fixed experimental pipelines, conflating recognition accuracy with descriptor robustness.

These two aspects classification performance and descriptor stability—are fundamentally distinct. Classification performance reflects end-to-end system behavior under specific experimental conditions, whereas descriptor stability refers to the consistency of feature representations when the same pattern is observed under repeated, controlled perturbations that approximate real acquisition variability.

Recent studies have increasingly recognized robustness and stability as primary evaluation criteria. While quaternion-based descriptors offer inherent advantages in multichannel representation and invariance, and fractional orders enable parameterized control over noise sensitivity and locality, robustness is still commonly evaluated indirectly via performance degradation under limited perturbation sets. Direct quantitative assessment of feature vector variability under structured perturbations remains insufficiently explored.

This limitation motivates the present work. We introduce a spatiotemporal pseudo-random framework for robustness-oriented evaluation of Quaternion Fractional Moment descriptors. Rather than treating feature extraction as a deterministic mapping, the proposed framework introduces controlled pseudo-random variability inspired by cryptographic image transformation methodologies (Faiq et al.,). This framework complements classification-based evaluation by enabling direct measurement of descriptor stability as an intrinsic property, independent of downstream classifier behavior.

3. Proposed Work

This section describes how Quaternion Fractional Moments are used in developing a spatiotemporal pseudo-random framework oriented towards analyzing geometric patterns in terms of their stability against varying conditions in quaternion-based patterns. While involved in enhancing classification accuracy as defined in their classic pipes, this new pipe focuses on achieving the more fundamental objective of assessing the stability and reliability of quaternion-based patterns against varying conditions based on geometric patterns that are slightly varying but still quite close to actual patterns as defined in such contexts. This is a crucial development as it presents a clear demarcation between what is defined in feature patterns and what is defined in other patterns. The different steps within the suggested framework based on quaternion fractional moments are shown in Figure 1. The suggested framework comprises five processing steps: image preprocessing and spatial segmentation, spatiotemporal pseudo-random motion, extraction of quaternion fractional moments, and, lastly, pattern recognition from a robustness point of view. For every processing step, there was a constant aim is to preserve geometric properties under controlled variation and systematically evaluate stability as a descriptor.

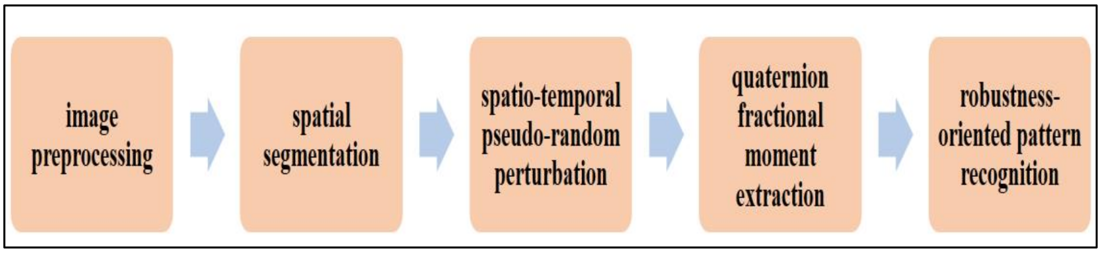

Figure 1. Structural Overview of the Proposed Framework

3.1 General Architecture of the Proposed Framework

The framework involves addressing a single input image using a set of pseudo-random cycles, where slight changes are introduced prior to Computing Quaternion Fractional Moments. The motivation for the approach is very straightforward: a representation should not change significantly if the underlying geometric pattern is unchanged since a robust representation remains unchanged when slight changes occur during normal functioning. A set of feature vectors is generated based on characterizing a geometric pattern, but this time using a different realization. The vectors are not viewed as holding a similar set of information; rather, they are put to empirical test concerning stability. By examining them altogether as a collection, one can then gauge robustness properties and determine which regimes of controlled variability are associated with enhanced sensitivity measures. Significantly, this framework is not intended for optimizing classification performance. Even when used as a secondary test method, the primary objective of classification may not be the objective of robustness testing. Since robustness testing and recognition accuracy are decoupled, the implication that high recognition accuracy necessarily implies high robustness measures is avoided.

3.2 Image Preprocessing and Spatial Segmentation

The particular reason for choosing Otsu's method for image thresholding is its ability to allow an objective procedure from subtracting a geometric design for the background. In practice, there would not be any direction for the geometric design in the analysis procedure. This method works well for Moroccan Andalusian geometric designs, which belong to the symmetry groups p4m and p6m. This is because the geometric properties of these designs not only provide aesthetic appeal but are also important in a manner that would not be altered in the analysis procedure, particularly in conducting image thresholding to determine the boundaries of these designs in the geometric plane. However, global thresholding would provide a marginal improvement in the properties of symmetry and boundaries for pixels.

The dataset used in this research is Moroccan Andalusian geometric tiles with p4m or p6m symmetry. Feature extraction is carried out after the preprocessing of the datasets. This is valid in the context of every image dataset, since no removal or decrease of common image artifacts is taken into consideration to provide a suitable dataset. All images are normalized to have a common spatial dimension, and additionally, color normalization is accomplished, involving a procedure for comparison of the descriptors among a few samples. Next, median filtering is applied, where impulsive noise is eliminated while, retaining the geometric edges, which are exceptionally clear and significant in the decision or assessment of symmetry among tiling features. This is achieved since noise is linked to image segmentation, which is primarily a thresholding process, specifically emphasizing a particular type of element segmentation, which is a design-irrelevant. A basic thresholding method might be sufficient for a specific type of segmentation, provided that a certain pixel intensity threshold is specified. Next, the segmented images are subjected to a second pseudo-random spatial transformation.

3.3 Spatiotemporal Pseudo-Random Perturbation Strategy

The central novelty of this work is the introduction of a spatiotemporal pseudo-random perturbation mechanism. Inspired by pseudo-random strategies used in image encryption, the mechanism injects controlled variability into the analysis while preserving the underlying structural integrity of the geometric motif. The point is not to distort the pattern beyond recognition, but to reproduce the kinds of small, routine disruptions that arise in real image acquisition and to examine whether the descriptor remains dependable when those disruptions occur.

3.4 Spatiotemporal Perturbation Model

At pseudo-random cycle t, the segmented image is transformed as

$$I_{t(x,y)} = T_{t(I(x,y))} \tag{1}$$

where $T_t = (\theta_t, s_t, \delta_t)$ with θt denoting the rotation parameter, s_t the scaling factor, and $\delta_t$representing localized perturbations. The parameters are generated using a pseudo-random sequence constrained within predefined bounds to ensure reproducibility.

3.4.1 Spatial Variability

Spatial variability is introduced by subjecting the segmented motif to pseudorandom transformations that are bounded by explicit ranges. In each cycle, the framework applies controlled rotations, mild scaling shifts, and localized perturbations. The parameters are therefore not left to drift. They are drawn from a pseudorandom process constrained to predefined intervals, so that the induced variability remains interpretable and does not collapse the geometric relations the analysis is meant to track. These spatial variations are intended to emulate the conditions under which ornaments are usually recorded outside controlled settings. Even when the underlying motif does not change, patterned surfaces are captured through small viewpoint shifts, slight geometric warping, imperfect framing, and minor irregularities produced by wear, repair, restoration traces, or uneven capture. By introducing this variability in a controlled manner, the framework turns an assumption into a measurable requirement. It tests whether the descriptor remains stable when the image is altered in the same modest, recurrent ways that real acquisition tends to introduce.

3.4.2 Temporal Variability

Temporal variability is introduced by treating feature extraction as an iterated procedure rather than as a single, definitive computation. The same input image is processed across multiple pseudorandom cycles, and each cycle uses a distinct but bounded configuration of spatial perturbations. The result is therefore not one representation but a sequence of feature vectors that all refer to the same underlying motif.

This repetition installs a temporal dimension in the evaluation. It allows the descriptor to be approached as a signature whose reliability is tested through recurrence. If the representation is robust, the vectors produced across cycles should cluster tightly, showing only limited dispersion. If the representation is sensitive, the vectors will drift from one cycle to the next. Robustness is thus assessed as stability under repeated, realistically varied extractions rather than inferred indirectly from fluctuations in downstream classifier performance.

3.4.3 Parameter Selection and Design Justification

The design parameters of the spatiotemporal pseudorandom framework were chosen to balance two demands that are often in tension. Robustness analysis requires sufficient repeated trials and enough controlled variability to justify stability claims. At the same time, the procedure must remain computationally tractable if it is to operate as an evaluation protocol rather than as a one-time demonstration.

A central control variable is the number of pseudorandom cycles T, which determines how reliably stability can be estimated. We tested $T \in \{5, 10, 20\}$. Although larger T increases statistical confidence, the gain becomes marginal once the feature distributions begin to converge. For that reason, T = 10 was selected as a pragmatic compromise. It provides a sufficiently dense set of realizations to observe stabilization in the descriptor's behavior, while avoiding the runtime overhead associated with T = 20, where improvements were limited relative to cost.

Perturbation bounds were selected to approximate realistic variability in cultural heritage imaging while preserving the crystallographic organization of the motifs. Rotation angles were restricted to ±15°, ±30°, and ±45°, which correspond to plausible misalignment introduced by camera tilt, framing differences, or digitization practices. Photometric variability was introduced through additive Gaussian noise with $\sigma \in \{0.01, 0.05\}$. These noise levels stress test descriptor sensitivity without collapsing the motif into an unstructured signal. Scaling was constrained to a narrow interval around unity, ($s \in [0.95, 1.05]$), which models minor distance and resolution changes without producing severe geometric warping.

Taken together, these design choices keep variability bounded, reproducible, and meaningful. The perturbations are strong enough to probe stability, yet constrained enough that the perturbed images remain recognizable instances of the same underlying pattern. This condition is essential because robustness assessment is only interpretable when variability remains within the space of routine acquisition conditions.

3.5 Quaternion Fractional Moment Extraction

Following spatiotemporal perturbation, Quaternion Fractional Moments are computed to obtain geometric descriptors. Quaternion algebra is used to represent color images holistically, which allows multichannel information to be encoded within a single mathematical object rather than being handled as independent channels.

3.5.1 Quaternion Image Representation

Let I(x,y) be a color image. In quaternion form, it is represented as

$$Q(x, y) = R(x, y)i + G(x, y)j + B(x, y)k \tag{2}$$

In this formulation, the RGB components are embedded into the imaginary part of the quaternion, forming a pure quaternion representation. This approach preserves the intrinsic correlations between color channels and provides a compact and holistic description of color information, which is well suited for quaternion-based moment analysis.

Based on this representation, fractional-order moments are subsequently applied to $Q(x,y)$ to obtain quaternion-valued coefficients that capture the global geometric structure of the image while remaining robust to spatiotemporal perturbations.

3.5.2 Quaternion Fractional Moments

The quaternion fractional moment of order $(\boldsymbol{n}, \boldsymbol{m})$, towards a securing of a given fractional order α, is defined in the follow way:

$$QFM_{n,m}^{(\alpha)} = \iint_{\Omega} Q(x,y)\, \Phi_{n,m}^{(\alpha)}\,(x,y)dxdy \quad (3)$$

where Ω denotes the image domain and $\Phi_{n,m}^{(\alpha)}\,(x,y)$ is the fractional basis function.

In polar coordinates (r, θ), the fractional basis function can be expressed as:

$$\Phi_{n,m}^{(\alpha)}\,(r,\theta) = R_n^{(\alpha)}(r)e^{-jm\theta} \quad (4)$$

where $R_n^{(\alpha)}(r)$ denotes the fractional radial function.

3.5.3 Interpreting the Fractional Order α

The fractional order α acts as a practical control parameter that shapes what the descriptor privileges. Lower α values emphasize global organization and the broad symmetry layout, which can improve stability under rotation, noise, and small imperfections. The trade off is reduced sensitivity to fine ornamental details that may distinguish motifs within the same symmetry family. Higher α values increase sensitivity to local structure and subtle differences, but they also increase susceptibility to disturbance, meaning that modest noise or partial degradation can produce larger shifts in feature values.

In this study, we set α = 0.8 because it repeatedly produced the most balanced behaviour for Moroccan Andalusian patterns governed by p4m and p6m symmetries. The value is not presented as universally optimal. It was selected empirically because it retained enough global regularity to remain robust under perturbations while preserving enough local sensitivity to differentiate motifs that share broad symmetry characteristics. This choice is also consistent with prior work on quaternion fractional moments, where intermediate fractional orders often provide improved stability under noise and moderate geometric variation.

The role of α is especially consequential in our setting because feature extraction is not performed once. Quaternion fractional moments are computed across multiple pseudorandom cycles, and each cycle yields a feature vector associated with a slightly different but controlled realization of the same image. Collecting feature vectors across cycles yields a distribution rather than a single point estimate. This is what allows robustness to be evaluated in concrete terms through the tightness of clustering, the magnitude of dispersion under controlled variability, and the degree of invariance when perturbations approximate routine acquisition conditions.

3.6 Robustness-Oriented Pattern Recognition

The extracted feature vectors are not treated primarily as inputs for a competition in accuracy. They are treated as empirical traces of how a descriptor behaves when the same motif is forced to reappear under routine, low amplitude instabilities of acquisition. The point is therefore robustness in the strict sense. It concerns whether the representation holds together when illumination shifts, when viewpoint drifts, when noise accumulates, and when the surface itself carries wear and minor irregularities. In this framework, the analysis is anchored in statistical behavior rather than in a single deterministic output. We compute intra class variance, inter class separability, and, crucially, the consistency of features across pseudorandom cycles.

For validation, conventional classifiers such as Support Vector Machines and ensemble based models can be introduced as secondary instruments. Their role is evidentiary, not constitutive. Classification performance does not define success here. It operates as supporting proof that the spatiotemporal pseudorandom strategy does not merely stabilize features in the abstract, but can also translate into improved recognition behavior when recognition is requested.

3.6.1 Feature Stability Metric

Let f_t denote the feature vector extracted at cycle t, and $\bar{\mathrm{f}}$ be the mean feature vector computed over T cycles. The descriptor stability is quantified as follows:

$$Stability = \frac{1}{T}\sum_{t=1}^{T} || f_t - \bar{\mathrm{f}} ||_2 \tag{5}$$

Lower values indicate higher stability under spatiotemporal variability.

Descriptor dispersion is then read quantitatively as the degree of deviation of feature vectors across cycles. In operational terms, dispersion corresponds to the average Euclidean distance between individual realizations and their mean representation. Low dispersion indicates that the descriptor reconstitutes the motif as the same object across perturbations, while high dispersion indicates that the descriptor is being pulled by capture noise and local disruptions. Unlike scalar variance, this stability metric is explicitly multi-dimensional. It registers dispersion in the quaternion feature space as a single, holistic measure of robustness, rather than as a set of disconnected fluctuations across independent coordinates.

3.7 Algorithm Description

Algorithm 1: Spatiotemporal Pseudorandom Quaternion Fractional Moment Analysis

Input a geometric pattern image

Apply preprocessing and Otsu-based segmentation

Initialize the pseudo-random generator with fixed bounds

For each cycle t = 1 to T:
- a. Apply controlled pseudo-random spatial perturbation
- b. Compute Quaternion Fractional Moments
- c. Store the resulting feature vector

Compute mean feature representation across cycles

Evaluate stability and dispersion metrics

Perform optional classification for validation

Output robustness and recognition results

3.8 Flowchart Description

Figure 2 presents the flowchart of the proposed framework. It summarizes the sequential execution of preprocessing, spatiotemporal perturbation, feature extraction, and robustness analysis. The diagram foregrounds the iterative structure of the pseudorandom cycles and makes explicitly describe the separation between robustness evaluation and classification-based validation.

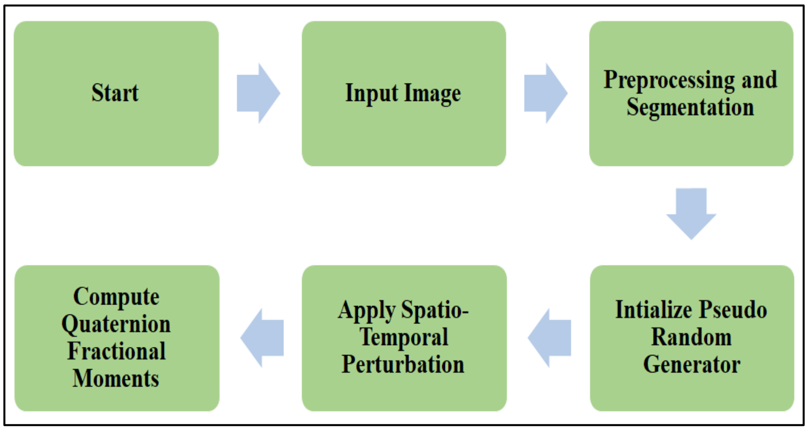

Figure 2. Flowchart of the Proposed Framework

3.9 Computational Complexity and Runtime Analysis

The computational cost of the proposed framework scales linearly with the number of spatiotemporal cycles T. The reason is structural. Each cycle repeats the same two operations, a bounded perturbation step followed by Quaternion Fractional Moment computation. For an input image of size N×M, the overall time complexity is therefore $O(T \times N \times M)$ since the full image domain is processed in each of the T cycles. A practical advantage is that the cycles are mutually independent. Each cycle yields a feature vector under its own pseudorandom perturbation, and no cycle requires intermediate results from any other. The framework is therefore naturally parallelizable. In practice, cycles can be distributed across CPU cores or executed on GPU hardware without altering the algorithmic outputs. As a result, increasing T to strengthen stability estimation does not necessarily imply a proportional increase in wall clock runtime, provided parallel execution is available.

4. Experimental Configuration

4.1 Dataset Description

A dataset of Moroccan Andalusian geometric designs with the crystallographic symmetry groups p4m and p6m is used for the experimental evaluation. The dataset includes 1,055 images in total, of which 669 belong to the symmetry group p4m and the remaining 386 to the symmetry group p6m. The images were chosen so that the evaluation is not limited to a small group of extremely similar patterns during the dataset selection process. The images have been normalized to a constant size before the features are extracted to rule out the possibility that variations in behavior are caused by the size of the input images. The dataset that the researchers collected for the study includes the most suitable Moroccan Andalusian designs, which have been selected from the most reliable design sources, including ornamental patterns found on buildings.

Table 1. Reports The Class Distribution and the Train Test Split Used in the Experiments

Class	Symmetry Group	Total Images	Training Set	Test Set
Class 0	p4m	669	534	135
Class 1	p6m	386	310	76
Total	—	1055	844	211

This table highlights the class imbalance and justifies the use of Macro-F1.

4.2 Evaluation Protocol

Next, using an 80:20 stratified split ratio, the dataset was split into training and testing datasets. The robustness evaluation used bounded perturbations to simulate the actual sources of variability in the production of cultural heritage images. These included using rotational tolerances (15°, 30°, and 45°) along with Gaussian noise (σ=0.01 and 0.05). The Support Vector Machine with the radial basis function kernel was used for the classification problems. Accuracy and Macro F1 were considered in evaluating performance. Since Macro F1 is more robust compared to Accuracy, specifically for imbalanced class distributions, its use is justified. Additionally, for comparison purposes among different methods, a metric evaluating stability with regard to spatiotemporal pseudo-random cycle length has been included.

4.3 Stability Metric

All feature vectors were L2 normalized before stability computation in order to produce a scale invariant robustness indicator. This is the definition of the normalized stability metric:

$$Stability_{Norm} = \frac{1}{T}\sum_{t=1}^{T} || \hat{f}_t - \bar{\hat{f}} ||_2 \tag{6}$$

where $\hat{f}_t$ denotes the normalized feature vector extracted at cycle t, and $\bar{\hat{f}}$ represents the mean normalized feature vector computed over T cycles. Lower values of $Stability_{Norm}$ indicate higher descriptor stability under spatiotemporal variability.

5. Results and Discussion

5.1 Baseline and QFM Performance

We first tested a classifier that used the Quaternion Fractional Moments extracted in a standard deterministic manner that is, without spatiotemporal pseudo-random variation in order to establish a baseline for our study. Therefore, the accuracy and the Macro-F1 measure were about 0.640 and 0.570, respectively. It should be mentioned that better results were obtained for the p4m symmetry, while the classification of p6m patterns continued to be more difficult. Given that hexagonal patterns typically support higher levels of structural complexity, this was to be expected.

Table 2. Performance Comparison Under Clean Conditions (No Rotation, No Noise)

Method	Accuracy	Macro-F1
Baseline QFM	0.640	0.570
Proposed QFM-PR (T=10)	0.630	0.559

The small trade-off in clean condition accuracy is easier to understand with this table. Along with precision and Macro F1.

Recognition error rate is defined as: $Error = 1 - Accuracy.$

The proposed QFM PR framework yields an error of 0.3697 under clean conditions, while the baseline QFM has an error of 0.3600. Therefore, the error is marginally higher when there is no perturbation. This result is in line with the design intent of the framework. QFM PR is not designed to peak in performance under ideal imaging conditions. Robustness is prioritized over accuracy in its design. A good selection among the representations that remain invariant, as opposed to those that exhibit significant variation for a single scenario, is ensured by introducing suitable variations in the feature extraction and information aggregation processes over cycles. As the description becomes less specific to the minute details of the clean image, this could lead to a slight loss of precision in clean images. However, once the distortions become realistic, as will be covered in the next point, this would be advantageous. The main idea of the proposed research work is that it would enable a slight loss of precision in clean images to be less affected by noise and smaller geometrical changes.

5.2 Robustness Under Rotation and Noise

Table 3 presents the results for the robustness conducted using the base QFM and the proposed spatiotemporal pseudorandom framework. For rotational conditions, both frameworks demonstrate the expected drop-off in accuracy, particularly at 45°. For Gaussian conditions, the new framework is more stable than the base framework. Specifically, QFM-PR achieves Macro-F1 values of 0.593 (σ=0.01), and 0.589 (σ=0.05), compared to 0.574 and 0.570, respectively, for the base.

All the comparison results shown in Table 3 were generated under the same experimental procedure. The test split, perturbation patterns, dimensionality of features, and SVM setup remained the same for both techniques. This consistency is important since the relative performance gains identified should correspond to the pseudo-random approach under test rather than changes in the testing procedure.

Table 3. Robustness Evaluation of Baseline QFM and the Proposed Spatiotemporal Pseudo-Random Framework Under Rotation and Gaussian Noise

Method	Perturbation	Rotation (°)	Noise (σ)	Accuracy	Macro-F1
Baseline-QFM	None	0	0.00	0.6398	0.5700
Baseline-QFM	Rotation	15	0.00	0.6635	0.5757
Baseline-QFM	Rotation	30	0.00	0.6588	0.5282
Baseline-QFM	Rotation	45	0.00	0.6398	0.4799
Baseline-QFM	Noise	0	0.01	0.6446	0.5737
Baseline-QFM	Noise	0	0.05	0.6398	0.5700
Proposed-QFM-PR (T=10)	None	0	0.00	0.6303	0.5587
Proposed-QFM-PR (T=10)	Rotation	15	0.00	0.6209	0.5660
Proposed-QFM-PR (T=10)	Rotation	30	0.00	0.6256	0.5329
Proposed-QFM-PR (T=10)	Rotation	45	0.00	0.6351	0.4606
Proposed-QFM-PR (T=10)	Noise	0	0.01	0.6588	0.5927
Proposed-QFM-PR (T=10)	Noise	0	0.05	0.6540	0.5889

These results demonstrate that the proposed PR strategy effectively stabilizes QFM descriptors against photometric noise by reducing sensitivity to random fluctuations.

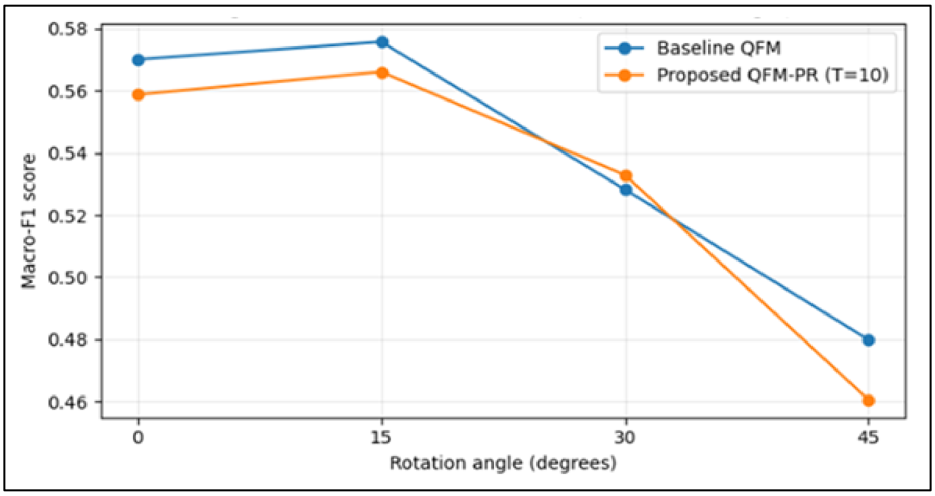

Figure 3. Macro-F1 Score Comparison between Baseline QFM and the Proposed QFM-PR (T = 10) Under Rotational Perturbations

Figure 3 shows the Macro F1 values for both the baseline QFM and the proposed QFM PR framework for T=10 for the rotated patterns. It can be observed from the plot that as the rotation angle increases, both curves monotonically decrease with the greatest degradation at 45°. This is due to that more rotated angles increase the geometrical difference and reduce the capability to predict the symmetry characteristics of the motif.

However, the role of the QFM PR is to ensure that the degradation effect caused by the rotation is tracked without any emphasis on the degradation trend, particularly when the rotations are intensified. This means that the degradation effect caused by rotation is not eliminated but it appears that the proposed framework has the ability to handle the rotational degradation effect more effectively. This is anticipated, considering that the aim of the proposed framework is based on this concept. The descriptor is made robust through the injection of a controlled pseudo-random perturbation throughout the process, which is a positive attribute, particularly when considering a certain scene from different angles.

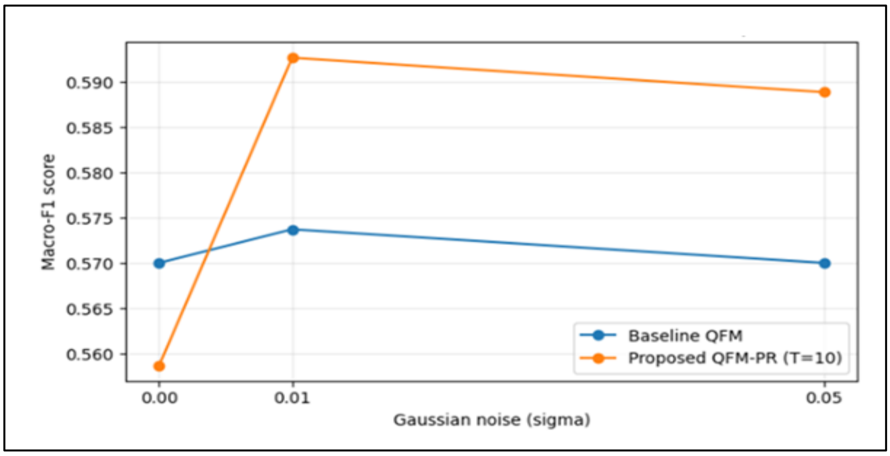

Figure 4. Macro-F1 Score Comparison between Baseline QFM and the Proposed QFM-PR (T = 10) Under Gaussian Noise Perturbations

Figure 4 shows the Macro F1 scores under increasing Gaussian noise. It can be observed that the baseline QFM is not affected much by the noise levels, exhibiting a slight decrease at noise levels σ=0.01 and σ=0.05. On the other hand, the QFM PR with T = 10 does not fall behind the Macro F1 values at both noise levels. This trend confirms the effectiveness of the spatiotemporal pseudorandom approach in enhancing the noise robustness of the QFM. Alternatively, the proposed approach resists the noise better compared to the deterministic approach, thereby justifying the assertion of stabilizing the QFM features with realistic levels of image noise.

5.3 Stability Analysis

Apart from the classification scores, the stability of descriptors was measured directly along the spatiotemporal pseudo-random cycles. A low normalized stability score of 0.042±0.010 is obtained by the proposed framework. This shows that the obtained QFM descriptors are strongly clustered together even for repeated extractions. This result is significant because it places robustness at a descriptor level, meaning that robustness is not considered an implicit outcome of any specific classification.

It is helpful to specify the differences between the proposed logic of evaluation, and the baseline. In a conventional QFM pipeline, robustness is typically inferred from the change in performance after perturbations, while the features are determined just once in a deterministic fashion. A change in accuracy or macro F1 scores makes instabilities observable 'downstream' within this setting. Variability is modeled explicitly. Stability is determined from the spread of these instances around the mean. Pseudo-random cycles are employed in a repetitive fashion to produce several realizations of the features from a single pattern. Notably, the spread is determined in the domain of the features, which is the relevant space, especially with the stability of the reliability of the descriptors functioning under typical variability of acquisition.

The implication of stability within the framework directly affects margin-based classifiers, such as SVMs, even though it is not the focus of evaluation. The intra-class variance is smaller and the clusters of samples from the same class are more compact when the features are more stable. Larger margin classifications and more reliable generalization performance in the face of noise result from this. Conversely, as features drift with perturbations, margins shrink and class overlap increases, leading to less reliable prediction performance. Within the confines of the framework's classifier-independent objective, the gain resulting from QFM-PR stability can provide a plausible explanation for its strong generalization performance under Gaussian noise.

The results of the experiment generally indicate a balanced trade-off. Because the system prioritizes robustness over maximum discrimination capability, small losses of peak accuracy are expected in clean environments. However, repeatability and robustness may be the limiting factors in the context of cultural heritage image reconstruction tasks. From this perspective, the experimental results on noisy images and the stability measure offer clear quantitative confirmation of the suggested method's capacity to reduce feature variability and enhance the robustness of the descriptors.

6. Conclusion

This paper studies the robustness of Quaternion Fractional Moment (QFM) descriptors for geometric pattern recognition under photometric noise. The descriptors should be stable and not peak classification accuracy in ideal scenarios. Thus, an evaluation framework is proposed that bounds spatiotemporal perturbations during feature extraction. A stability metric measures the robustness of a descriptor in the feature space independently of the classifier. Based on the results from Moroccan-Andalusian geometric patterns using QFM under Gaussian noise, these descriptors exhibit a more stable performance metric under recognition than other deterministic methods. According to the testing, robustness in the cultural heritage image collection and other areas under investigation should be prioritized. Additional perturbations and stability measurements should be included in future studies to further demonstrate the importance of stability analysis in evaluating these visual descriptors used for pattern recognition.

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