

Investigations on the Performance Comparison of Co-Prime Array with and without interpolation for DOA Estimation

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Abstract

The sparse arrays have recently attracted a lot of attention in the last decade, due to their ability to find more sources than sensors present in an array configuration. There might be some sparse arrays which might have holes in their difference coarray, and different interpolation techniques are used to fill these holes. In this paper, the comparison of estimation performances using RMSE vs SNR plots of co-prime array before and after interpolation are analyzed using both coarray MUSIC and ESPRIT algorithms.

Keywords: sparse arrays, Co-Prime Array (CPA), holes, difference coarray, interpolation techniques, coarray MUSIC, ESPRIT.

1. Introduction

With the emergence of sparse arrays such as nested arrays, coprime arrays [1], etc. in the last decade, there has been a rejuvenated interest in DOA estimation problem [13,14,15]. The DOA estimation is important and is a challenging problem in domains such as radar, sonar, wireless communications, etc. The subspace algorithms such as coarray MUSIC [2] and ESPRIT [3] can be used to estimate the performance of such sparse arrays.

There are some sparse arrays that might have holes; for example, the Co-Prime Arrays (CPA) for $M=3$ and $N=5$ configuration, the Difference coarray D contains sensors from -25 to $+25$ with sensors missing at $-24, -23, -21, -18, 18, 21, 23$, and 24 which are known as holes. This means that there is a continuous Uniform linear array segment U from -17 to $+17$, and these holes are filled by various techniques [4,5,6,7] to further improve the Degrees of Freedom (DoF) [8], thereby improving the detection of more number of sources. There has been research going on in the direction interpolation of difference coarray [9] and in the direction of mutual coupling also.

There are some previous works that exist in literature, that have addressed the concept of array interpolation. In [4], a k-times extended CPA configuration was introduced to achieve larger number of DoF which will fill these holes. In [5], two novel arrays for co-prime array was proposed, which were able to increase DoF without being overly sensitive to mutual coupling. In [6], reducing sensor spacing of original antenna array, construction of steering matrix for the virtual array and also using virtual array vector for the construction of co-variance matrix were performed. In [7], a structured matrix completion method based on Semi Definite Programming was used to fill the co-variance matrix which improves probability of resolution and accuracy. In [8], nuclear norm minimization was used to fill the holes in CPA, and then the co-variance matrix was calculated based on virtual array. Using coarray MUSIC algorithm, the accuracy is found to be improved. However, the following aspects have not been addressed in the above papers: filling of holes in CPA using l1-minimization, comparison of coarray MUSIC and ESPRIT to determine which algorithm has better DOA estimation capability when used for sparse arrays, comparison of CPA along with Minimum Redundancy Array (MRA), and Nested Array (NA) using co-array MUSIC and ESPRIT. These aspects have been addressed in this paper.

The paper is organized in the following manner. The preliminaries that are required to understand the concepts of sparse array are introduced in section 2. Section 3 discusses the concepts of l1-minimization, difference coarray, central ULA, and smallest ULA. In section 4, the simulation results for the estimation performance of coprime array with or without interpolation have been illustrated. The section 5 elaborates the simulation results, and the conclusions that are drawn from these simulations are briefed in section 6.

2. Preliminaries

2.1 Signal Model

It is assumed that ‘q’ number of far-field, narrowband, uncorrelated, coherent, circular source signals are impinged on the Sparse Linear Array consisting of ‘p’ sensors, where the sensors are separated from the distance ‘nd’, from the DOAs $\theta_1, \theta_2, \dots, \theta_q$. Here the distance is non-uniform ($d = \frac{\lambda}{2}$ minimum distance between the sensors), and ‘n’ is non-linear in some integer set $\mathbb{S}$. The Measurement Vector (or) Received Signal Vector $\mathbf{x}(t) \in \mathbb{C}^{p \times 1}$ can be modelled as,

$$\mathbf{y}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{n}(t) \quad (1)$$

where, 't' indicates time index, $x(t) \in \mathbb{C}^{p \times 1}$ indicates the signal waveform, $n(t) \in \mathbb{C}^{p \times 1}$ indicates the additive white Gaussian noise, and $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_q)] \in \mathbb{C}^{p \times q}$ indicates Array Manifold Matrix and the array.

$$\mathbf{a}(\theta_q) = \begin{bmatrix} e^{j2\pi \frac{p_1}{\lambda} \sin(\theta_q)} & e^{j2\pi \frac{p_2}{\lambda} \sin(\theta_q)} & \dots & \dots & e^{j2\pi \frac{p_N}{\lambda} \sin(\theta_q)} \end{bmatrix}^T \quad (2)$$

Eq (2) indicates the steering vector corresponding to θ_q , and the 'p' sensor positions are denoted by $p_1, p_2, \dots, p_p$, where $p_1, p_2, \dots, p_p$ are the elements in the Set $\mathbb{S}$.

The array covariance matrix can be approximated as,

$$\mathbf{R}_{yy} = E[\mathbf{y}(t)\mathbf{y}(t)^H] = \mathbf{A}\mathbf{R}_{qq}\mathbf{A}^H + \sigma_p^2 \mathbf{I}_p^2 \quad (3)$$

where, $\mathbf{R}_{qq} = E[\mathbf{x}(t)\mathbf{x}(t)^H]$ is the source signal covariance matrix, σ_p^2 is the noise power and $\mathbf{I}_p^2$ denotes the 'p*p' identity matrix. In practice, the array covariance matrix for 'r' number of snapshots can be approximated as,

$$\hat{\mathbf{R}}_{yy} = \frac{1}{r} \sum_{t=1}^r \mathbf{y}(t)\mathbf{y}(t)^H \quad (4)$$

2.2 COARRAY MUSIC AND ESPRIT

Here, the new Hermitian Toeplitz matrix is $\tilde{\mathbf{R}}_{\mathbb{U}}$ is formed from the following finite snapshot auto correlation vector $\hat{\mathbf{x}}_{\mathbb{U}}$ [10].

$$\langle \tilde{\mathbf{R}}_{\mathbb{U}} \rangle = \langle \hat{\mathbf{x}}_{\mathbb{U}} \rangle_{n_1, n_2}, \tilde{\mathbf{R}}_{\mathbb{U}} \in \mathbb{C}^{|\mathbb{U}^+| \times |\mathbb{U}^+|} \quad (5)$$

where, $\mathbf{n}_1, \mathbf{n}_2 \in |\mathbb{U}^+| = \{n | n \in \mathbb{U}, n \geq 0\}$. Applying MUSIC algorithm [11] on $\tilde{\mathbf{R}}_{\mathbb{U}}$ is same as applying MUSIC on covariance matrix of received data. The auto correlation vectors and finite snapshot autocorrelation vectors defined on the $\mathbb{D}, \mathbb{U}, \mathbb{V}$ are $\mathbf{x}_{\mathbb{D}}, \mathbf{x}_{\mathbb{U}}, \mathbf{x}_{\mathbb{V}}$ and $\tilde{\mathbf{x}}_{\mathbb{D}}, \tilde{\mathbf{x}}_{\mathbb{U}}, \tilde{\mathbf{x}}_{\mathbb{V}}$ respectively. The ESPRIT algorithm operates on signal subspace in a similar way as MUSIC algorithm operates on noise subspace. ESPRIT algorithm [12] is applied on $\tilde{\mathbf{R}}_{\mathbb{U}}$ to estimate the DOAs of the corresponding signals.

3. Coarray Interpolation using L-1 Minimization

3.1 L1-Minimization

Here, l-1 minimization is used to interpolate the finite snapshot autocorrelation vector $\tilde{\mathbf{x}}_{\mathbb{D}}$ to $\tilde{\mathbf{x}}_{\mathbb{V}}$. Then, the vector $\tilde{\mathbf{x}}_{\mathbb{V}}$ is used to construct the $\tilde{\mathbf{R}}_{\mathbb{V}}$, where $\tilde{\mathbf{R}}_{\mathbb{V}}$ has toeplitz kind of structure.

3.2 Co-prime Array

The CPA consists of two sparse Uniform Linear arrays. Coprime arrays are characterized by two numbers M and N, where the first array consists of N elements with separation M and the second subarray consists of 2M-1 elements with N separation. The CPA for M=3 and N=5 is mentioned in Fig.1.

$$\mathbb{S}_{cop} = \{0, M, 2M, \dots, (N-1)M, N, 2N, \dots, (2M-1)N\} \quad (6)$$

where, $\mathbb{S}_{cop}$ denotes the physical sensors or physical element locations. The number of physical elements present in coprime array are given by 2M+N-1 elements [12].

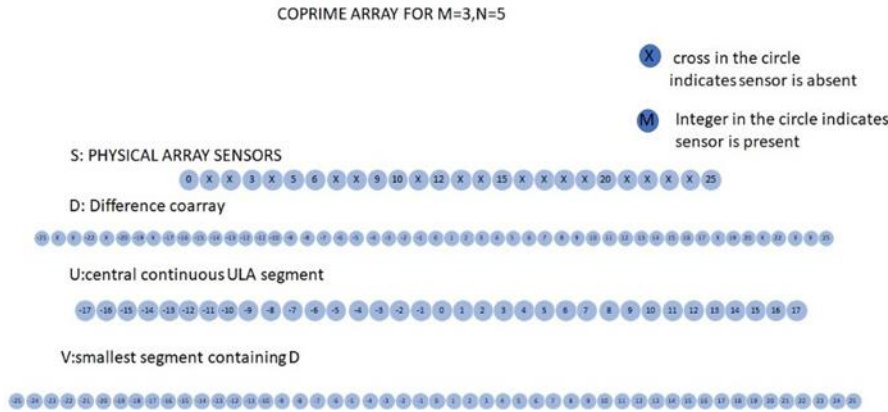


Figure 1. CPA for M=3 and N=5 configuration

3.3 DIFFERENCE COARRAY

The difference coarray ($\mathbb{D}$) [12] for any specified sparse array denoted by $\mathbb{S}$, which are integers, is specified as,

$$\mathbb{D} = \{p_1 - p_2 | p_1, p_2 \in \mathbb{S}\} \quad (7)$$

3.4 CENTRAL ULA

The central continuous Uniform Linear Array segment [12] in the difference coarray is denoted by $\mathbb{U}$.

$$\mathbb{U} = \{m| -|m|, \dots, -1, 0, 1, \dots, |m| \in \mathbb{D}\} \quad (8)$$

3.5 Smallest ULA

The smallest ULA ($\mathbb{V}$) that contains the difference coarray $\mathbb{D}$, is given by the following equation.

$$\mathbb{V} = \{m| -\min \mathbb{D} \leq m \leq \max \mathbb{D}\} \quad (9)$$

3. Numerical Examples

The CPA consisting of $M=3$ and $N=5$ configuration is considered for the simulation, hence $|\mathbb{S}|=10$, $|\mathbb{D}|=43$, $|\mathbb{U}|=35$, $|\mathbb{V}|=51$. The detection of maximum sources detected by this coprime array configuration using coarray MUSIC is 17 [12]. Two cases are considered.

Case 1: BEFORE INTERPOLATION: In the first case i.e., before interpolation, the estimation performance of CPA with $M=3$ and $N=5$ configuration for ‘ q ’=15, i.e., the number of considered sources is less in number than the maximum number of detectable sources ($q=17$). The number of times the samples are being captured (or simply snapshots) is ‘ r ’=500 and the number of times the experiment is repeated as a whole (or simply Monte Carlo simulations) is ‘ K ’=100.

Case 2: AFTER INTERPOLATION: In the second case i.e., after interpolation, ‘ q ’=20 i.e., here the number of sources is more than the maximum detectable sources, ‘ r ’=500, and ‘ K ’=100. Here, the interpolation is done using l-1 minimization. For the first case, i.e., before interpolation, the spectrum plots are shown in Fig.2 and Fig.3, which illustrate the performance of CPA for $M=3$ and $N=5$ configuration for 15 sources and 20 sources using coarray MUSIC algorithm before interpolation.

As $|\mathbb{U}|=35$, the maximum sources that can be detected by this CPA configuration is equal to $\frac{|\mathbb{U}|-1}{2}=17$. From the Fig.2 and Fig.3, it can be inferred that estimation performance of coprime array using coarray MUSIC algorithm for 15 sources is way better than estimation performance of coprime array for 20 sources, since the spectrum peaks are not on the grid lines in Fig.3.

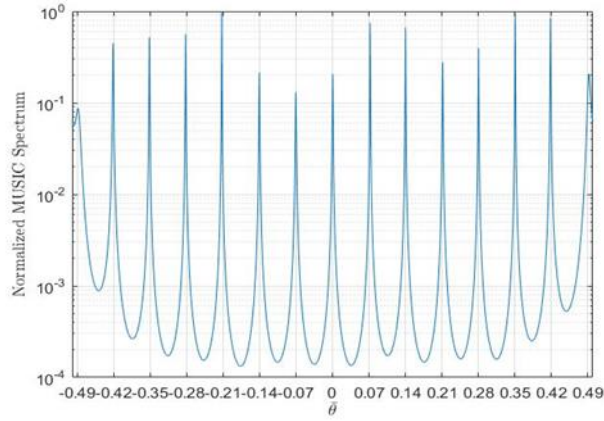


Figure 2. Spectrum plot of CPA using Coarray MUSIC algorithm for 15 sources before interpolation

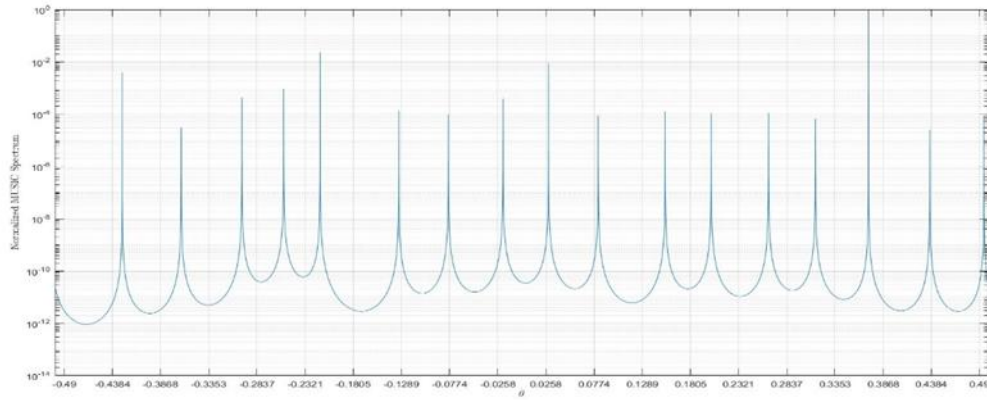


Figure 3. Spectrum plot of CPA using Coarray MUSIC algorithm for 20 sources after interpolation

where, $\mathbf{K}$ is the number of times the experiment is repeated (MCRUN), $\hat{\theta}_{j,q}$ is the estimate value at j^{th} MC run for q^{th} source, and θ_q – true angle for q^{th} source.

Fig.4 and Fig.5 illustrate the comparison of the performances, RMSE vs SNR of CPA with Nested Array $N_1=5$, $N_2=5$ sensors, Minimum Redundancy Array of 8 sensors, with the sensors at the locations $\mathbf{S}_{MRA} = \{0, 1, 4, 10, 16, 18, 21, 23\}$ [12] using both coarray MUSIC algorithm and ESPRIT algorithm for ‘r’ =500 snapshots and ‘K’ =100 Monte Carlo simulations. The length of finite snapshot autocorrelation vector i.e., $\tilde{\mathbf{x}}_U$ for the following coprime array, nested array and MRA considered in this paper are 35, 59, and 47 respectively,

and the maximum sources detected by the mentioned arrays are 17, 29, and 23 respectively. Hence, the performance metric RMSE for CPA at all considered SNR, before interpolation i.e., the first case, is less compared to the performances of both NA and MRA. Hence, the coprime array is less capable in handling estimation than NA and the considered MRA.

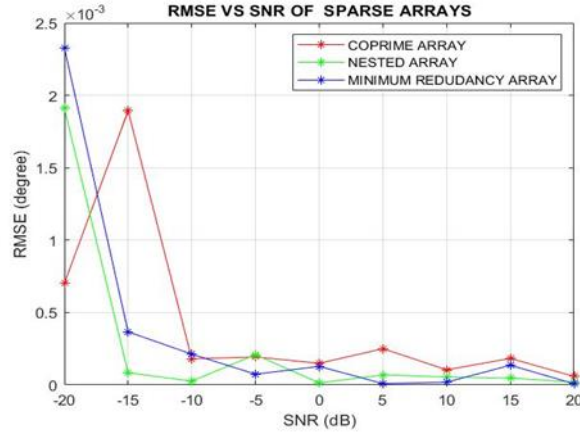


Figure 4. Comparison of RMSE vs SNR of CPA with NA and MRA using coarray MUSIC algorithm before interpolation

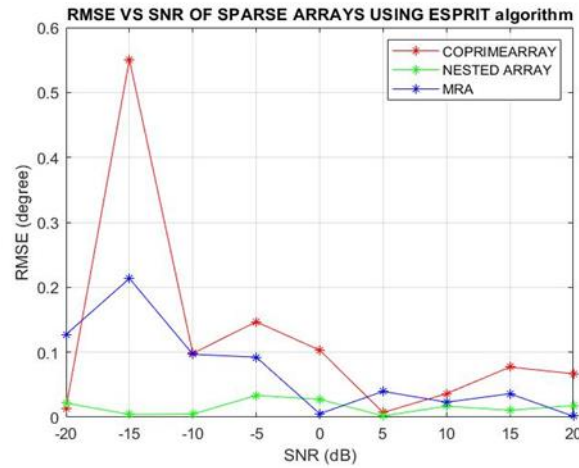


Figure 5. Comparison of RMSE vs SNR of CPA with NA and MRA using ESPRIT algorithm before interpolation

Fig.6 illustrates the comparison of performances, RMSE vs SNAPSHOTS of CPA with nested array $N_1=5$, $N_2=5$ sensors and MRA of 8 sensors using coarray MUSIC algorithm. In this illustration also, the estimation metric RMSE of coprime array, continues to remain less than the performance of NA and MRA, even when more number of samples are being captured.

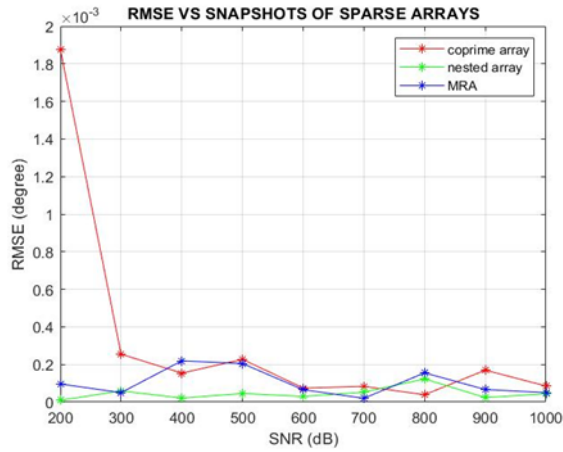


Figure 6. Comparison of RMSE vs SNAPSHOTS of CPA with NA and MRA using coarray MUSIC algorithm before interpolation

Fig. 7 illustrates the comparison of performances, RMSE vs SNAPSHOTS of CPA with nested array $N_1=5$, $N_2=5$ sensors and MRA of 8 sensors using ESPRIT algorithm. In this illustration also, the DOA estimation performance of coprime array is less than the performance of NA and MRA.

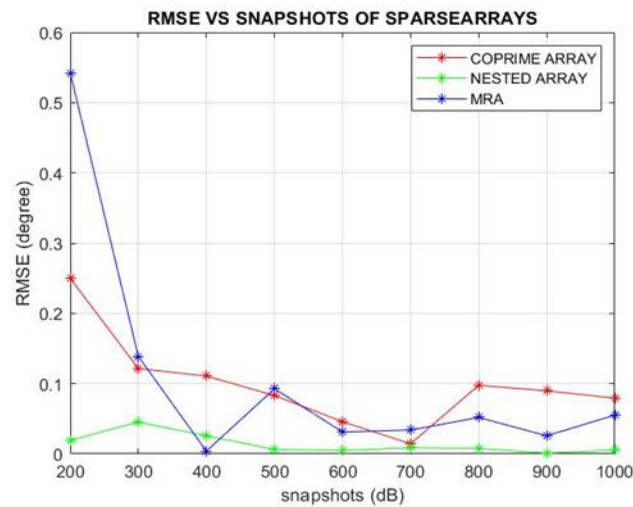


Figure 7. Comparison of RMSE vs SNAPSHOTS of CPA with NA and MRA using ESPRIT algorithm before interpolation

Now if the number of sources is increased to be greater than 17, then there is a decrease in the estimation performance to compensate for the estimation performance. The holes in the coprime array are filled using l-1 minimization technique, so that the maximum no. of detectable sources can be increased to 21 i.e., $(\lfloor \mathbb{D} \rfloor - 1)/2 = 21$ but not 25. This is because, even if all the holes are filled, $(\lfloor \mathbb{V} \rfloor - 1)/2 = 25$ cannot be achieved for the coprime array, because the actual DoF is commanded by the difference coarray $\mathbb{D}$. In the second case, the interpolation of holes is performed using l1-minimization, thereby maximizing the number of detectable

sources for the considered coprime array. Fig.8 shows that the peak lines are exactly on the grid, i.e., co-prime array is capable of detecting 20 sources after interpolation, using coarray MUSIC algorithm, i.e., coprime array estimation capability has been increased through interpolation.

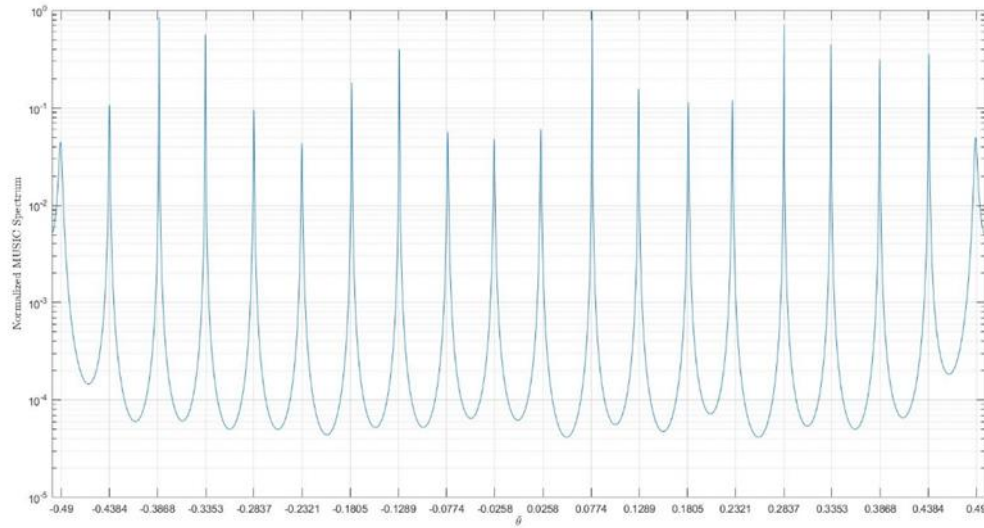


Figure 8. Spectrum plot of CPA for $M=3$ and $N=5$ after interpolation using coarray MUSIC algorithm

Fig.8 illustrates the spectrum plot of CPA after interpolation for 20 sources. Figures 3 and 8 can be compared, where the performance of coprime array is for 20 sources in both cases i.e., before interpolation and after interpolation RMSE vs SNR of coprime array with nested array and MRA using coarray MUSIC algorithm. Fig.9 illustrates the comparison of performances, RMSE vs SNR of coprime array with nested array and MRA using ESPRIT algorithm after interpolation. When comparing with Fig.4, there has been a significant improvement in Fig.9 as it is able to detect 20 sources with less RMSE compared to 15 sources in Fig.4.

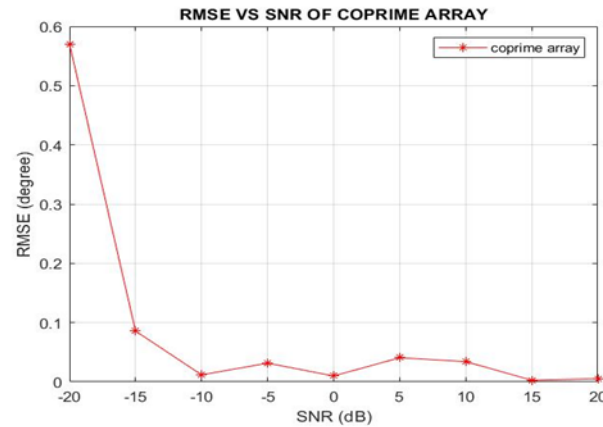


Figure 9. RMSE vs SNR plot for CPA for M=3 and N=5 after interpolation using ESPRIT algorithm.

Fig.10 and Fig.11 illustrate the comparison of performances, RMSE vs SNR of coprime array with NA and MRA using coarray MUSIC algorithm and ESPRIT algorithm after interpolation. The estimation performance of coprime array has been considerably improved and has been better than MRA by using both coarray MUSIC and ESPRIT algorithms, after interpolation.

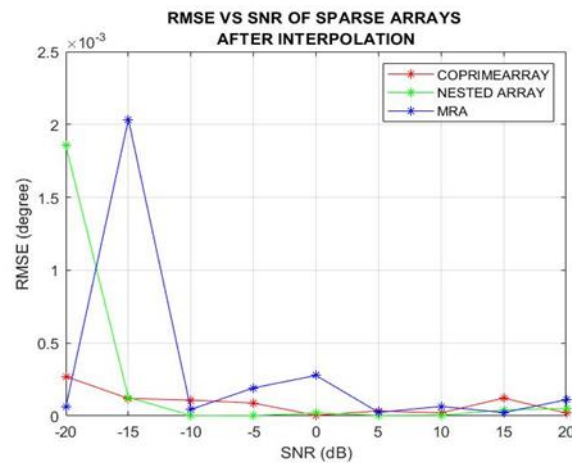


Figure 10. RMSE vs SNR comparison of CPA with NA and MRA after interpolation using coarray MUSIC algorithm

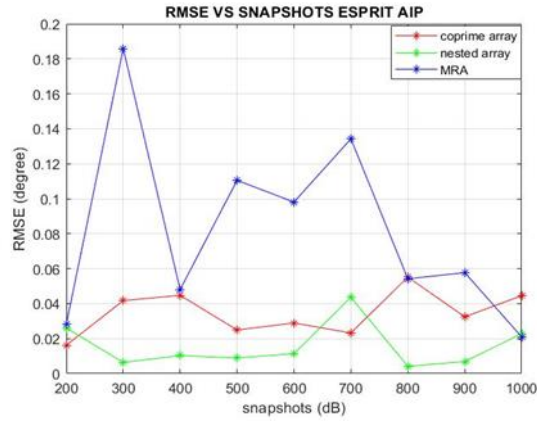


Figure 11. RMSE vs SNR comparison of CPA for M=3 and N=5 with NA and MRA after interpolation using ESPRIT algorithm

Fig.12 illustrates the comparison of performances, RMSE vs SNAPSHOTS of coprime array with nested array and minimum redundancy array using coarray MUSIC algorithm. Fig.13 illustrates the comparison of performances, RMSE vs SNAPSHOTS of CPA with NA and MRA after interpolation using ESPRIT algorithm. From Fig.12 and Fig.13, it can be concluded that the coprime array has better performance than MRA after interpolation.

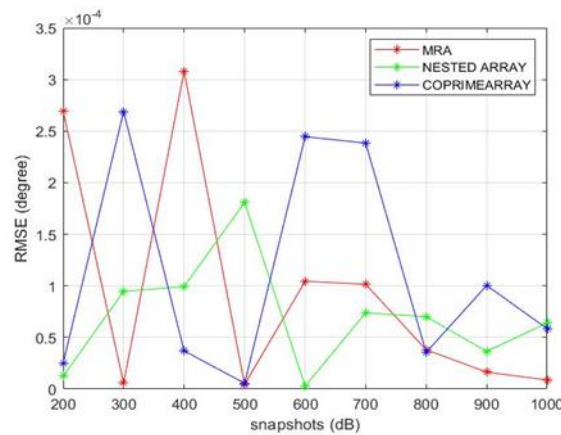


Figure 12. RMSE vs SNAPSHOTS comparison of CPA for M=3 and N=5 with NA and MRA by coarray MUSIC after interpolation

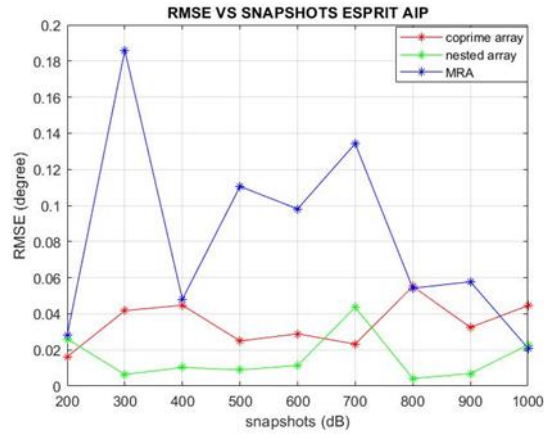


Figure 13. RMSE vs SNAPSHOTS comparison of CPA for M=3 and N=5 with NA and MRA by ESPRIT algorithm after interpolation

4. Results and Discussion

Table I and Table II explain the results obtained in the above numerical examples for RMSE vs SNR and RMSE vs SNAPSHOTS.

Table I: RMSE vs SNR at SNR = 0 dB

S.No.	Array Type	COARRAY MUSIC		ESPRIT	
		Before Interpolation ($\times 10^{-3}$)	After Interpolation ($\times 10^{-3}$)	Before Interpolation	After Interpolation
1.	CPA	0.30	0.08	0.1	0.01
2.	MRA	0.27	0.27	0.05	0.05
3.	NA	0.05	0.05	0.02	0.02

From Table I, using RMSE (vs) SNR plots at SNR = 0dB, the CPA has been able to estimate better than MRA, after interpolation using coarray MUSIC algorithm as illustrated in Fig.10. The CPA's estimation capability is better than MRA and NA, after interpolation using ESPRIT algorithm as illustrated in Fig.11. The estimation capabilities

of CPA, MRA, and NA have been better when the estimations are performed using coarray MUSIC algorithm than ESPRIT algorithm.

Table II: RMSE vs SNAPSHOTS at 500 SNAPSHOTS

S.No.	Array Type	COARRAY MUSIC		ESPRIT	
		Before Interpolation ($\times 10^{-3}$)	After Interpolation ($\times 10^{-4}$)	Before Interpolation	After Interpolation
1.	CPA	0.210	0.05	0.11	0.03
2.	MRA	0.195	0.10	0.09	0.11
3.	NA	0.015	1.75	0.02	0.01

From Table II, using RMSE vs SNAPSHOTS plots where number of snapshots =500, the CPA has been able to estimate better than MRA and NA, after interpolation using coarray MUSIC algorithm as illustrated in Fig.12. The CPA's estimation capability is better than MRA, after interpolation using ESPRIT algorithm as illustrated in Fig.13. The estimation capabilities of CPA, MRA, and NA have increased by order of 10 (i.e., RMSE decreased by order of 10) when estimations are performed using coarray MUSIC algorithm than ESPRIT algorithm.

5. Conclusion

The contribution of the paper includes estimation capability of CPA before and after interpolation, and how the estimation capability is improved further with both MUSIC and ESPRIT algorithms. Moreover, the estimation capability of CPA is compared with MRA and NA. All the simulations related to RMSE show that the estimation capability of CPA with help of coarray MUSIC algorithm has achieved good estimation results than ESPRIT algorithm, and after interpolation the coprime array has been able to detect more sources than before interpolation, and the performance of CPA after interpolation is improved than MRA as compared to before interpolation.

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