

Optimal Time for Withdrawal of Voluntary Retirement Scheme with a Probability of Acceptance of Retirement Request

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Abstract: Voluntary retirement schemes are observed in certain sectors of the Indian Industry such as Banking and Insurance. There is a need to determine the optimal time to withdraw a voluntary retirement scheme (VRS), balancing the reduction of high compensation of a segment of employees, prevent mass voluntary turnover from the grade, achieve optimal productivity and ensure normal business operations. In this paper an approach to derive an optimal policy to withdraw a voluntary retirement scheme considering a probability of acceptance of a retirement request, cost due to announcing a voluntary retirement scheme and cost to the organisation due to one-time special payments to those who retire during the time period is discussed.

Keywords: Voluntary retirement schemes, Optimal withdrawal time, Manpower planning.

Introduction

In certain service industry sectors, voluntary retirement (voluntary early retirement) schemes are announced periodically where employees could opt for early retirement. In today's economic scenario, government organizations in India are offering voluntary retirement schemes periodically with a view to disinvestment of its public sector undertakings.

The benefits to employees is probably that a) the compensation package is identical to that when the employee retires on the date provided by a normal employee contract, b) the preference for leisure is greater than that of an active job in the organisation, c) the employee's health condition forces him/her to opt for early retirement and/ or d) the labour market provides a better opportunity. However, there is a possibility that there is no alternate employment available due to market conditions or due to lack of adequate match of opportunities with respect to skills and seniority.

The benefit to the organisation is that a) the schemes provide an opportunity to replace highly paid employees with junior employees so that there is a saving on the total employee cost, b) there is a need to weed sub-productivity

and/ or c) there is a need to cut costs during the short-term. The organisation runs a risk of large recruitment and compensation costs when there is a critical need for skilled employees in the long-term.

Employee retirement is considered as a component of wastage in manpower planning [1]. Mandatory retirement is the fixed age at which people who hold certain jobs are required by industry custom or law to leave their employment or retire. Both mandatory and voluntary retirement schemes are ways to handle decline in employee's productivity [2]. An involuntary ending of an employee's career due to a layoff, disability or health problem is involuntary retirement. Also, based on an analysis of voluntary and involuntary early retirement on international microdata covering 19 industrialised countries, the authors consider voluntary retirement to be a preference for leisure as an alternative to continuing work [3]. References [4] and [5] have explicitly stated the difference between voluntary and involuntary early retirement. The optimal time to withdraw a voluntary retirement scheme has been proposed where every request for voluntary retirement is accepted [6]. The optimal withdrawal time for voluntary retirement scheme has been determined when the threshold level of the reserve level of an organization's finances varies with respect to time [7].

There is a need to determine an optimal policy to withdraw voluntary retirement schemes balancing the benefits due to reduction of high compensation of employees in specific grades against the impact of loss in work-life balance for the remaining employees. There is also a need to control mass voluntary turnover. A probability of acceptance of a voluntary retirement scheme request has been introduced to control mass voluntary turnover. In this paper, an approach to determine the optimal time to withdraw a voluntary retirement scheme, considering a probability of acceptance of a retirement request, cost due to announcing a voluntary retirement scheme and cost to the organisation due to one-time special payments to those who retire during the time period is discussed.

Model Description and Analysis

Model Description

It is assumed that voluntary retirement schemes are announced for a specific grade. Let the initial grade size be N . Voluntary retirement schemes are announced by an organisation typically when the grade size is large and when there are opportunities to reduce employee cost from a specific grade; further, they will be offered only for a specific time period, after which the scheme will be withdrawn irrespective of the number of persons who have opted for the scheme. It is assumed that only $(N - R)$ persons are needed in the grade. Hence the voluntary retirement scheme would be withdrawn when R persons are permitted to retire or when the specified time has elapsed. It is assumed that a person who opts for voluntary retirement will be permitted to retire with probability p and not be permitted to retire with probability $q = 1 - p$. The scheme is considered to be a failure if less than R persons retire.

Notations

The following notations are adopted in the analysis.

k : Number of epochs in $(0, T)$ in which voluntary retirement is permitted for the grade.

X_i : Number of persons retiring at the i^{th} epoch (discrete random variable).

$S_k = X_1 + X_2 + \dots + X_k$: the total number of persons retiring upto the k^{th} epoch.

$V_k(T)$: Probability that voluntary retirement scheme is offered in k epochs during $(0, T)$.

L : The number of persons who opt for retirement in k epochs.

R : The threshold level or the minimum number of employees who must retire voluntarily.

p : The probability that the voluntary retirement request by an employee is accepted by the organisation. $0 < p < 1$.

q : The probability that the voluntary retirement request by an employee is not accepted by the organisation. $q = 1 - p$.

C_v : Cost of voluntary retirement per employee in each of the epochs.

C_f : Cost of failure of the scheme. The base cost that must be incurred irrespective of the number of voluntary retirements in each epoch.

$f(\cdot)$: The probability density function of the inter-arrival time between epochs.

$f^{(k)}(\cdot)$: k -fold convolution of $f(\cdot)$.

$F(\cdot)$: Cumulative distribution function corresponding to $f(\cdot)$.

$F^{(k)}(\cdot)$: k-fold convolution of $F(\cdot)$.

T_c : Total cost of voluntary retirements in $(0, T)$.

While X_1 follows a binomial distribution with parameters N and p , X_2 conditional on X_1 will follow a binomial distribution with parameters $N - X_1$ and p , X_3 conditional on X_1 and X_2 will follow a binomial distribution with parameters $N - (X_1 + X_2)$ and p , and so on. Therefore, the sum S_k follows a binomial distribution with parameters N and $(1 - q^k)$.

Model Analysis

The total cost is incurred on account of two factors:

1. If the total number of persons who retire is less than the threshold level, then the scheme is deemed a failure and the cost incurred is the cost of retirement of the retirees along with the cost of failure of the scheme.
2. If the total number of persons who retire equals the threshold R , then the scheme is deemed a success and the total cost of the voluntary retirement scheme is the cost of retirement of the R retirees.

The total cost incurred is given by:

$$T_c = \sum_{k=1}^{\infty} V_k(T) \{ (P(S_k < R)) [C_F + C_v S_k] + (P(S_k = R)) C_v R \} \quad (1)$$

$$T_c = \sum_{k=1}^{\infty} [F^{(k)}(T) - F^{(k+1)}(T)] \left\{ \sum_{l=0}^{R-1} P(S_k = l) [l C_v + C_F] + P(S_k = R) R C_v \right\}$$

$$T_c = \sum_{k=1}^{\infty} [F^{(k)}(T) - F^{(k+1)}(T)] \left\{ C_v E[S_k] + C_F \sum_{l=0}^{R-1} P(S_k = l) \right\}$$

For an optimal T_c , $\frac{dT_c}{dT} = 0$. Therefore,

$$0 = \sum_{k=1}^{\infty} [f^{(k)}(T) - f^{(k+1)}(T)] \left\{ C_v E[S_k] + C_F \sum_{l=0}^{R-1} P(S_k = l) \right\} \quad (2)$$

$$0 = \sum_{k=1}^{\infty} [f^{(k)}(T) - f^{(k+1)}(T)] \left\{ C_v N (1 - q^k) + C_F \sum_{l=0}^{R-1} \binom{N}{l} (1 - q^k)^l (q^k)^{N-l} \right\} \quad (3)$$

T satisfying the above equation is the optimal time T^* . The corresponding total cost is the optimal expected cost T_c^* .

Special Case

It is assumed that the voluntary retirement scheme is offered at Poisson epochs.

Then,

$$f^{(k)}(T) = \frac{\lambda^k T^{(k-1)} e^{-\lambda T}}{(k-1)!}, \quad f^{(k+1)}(T) = \frac{\lambda^{k+1} T^k e^{-\lambda T}}{k!}$$

Also,

$$\sum_{k=1}^{\infty} [f^{(k)}(T) - f^{(k+1)}(T)] = \lambda e^{-\lambda T}$$

Equation (3) now becomes,

$$NqC_v e^{\lambda Tq} - N C_v e^{\lambda Tq} - C_F \sum_{k=1}^{\infty} \frac{(\lambda T)^{(k-1)}}{(k-1)!} \sum_{l=0}^{R-1} \binom{N}{l} (1-q^k)^l (q^k)^{N-l} + C_F \sum_{k=1}^{\infty} \frac{(\lambda T)^k}{k!} \sum_{l=0}^{R-1} \binom{N}{l} (1-q^k)^l (q^k)^{N-l} = 0$$

Therefore,

$$N \frac{C_v}{C_F} p e^{\lambda Tq} = \sum_{l=0}^{R-1} \binom{N}{l} \sum_{m=0}^l \binom{l}{m} (-1)^{l-m} e^{\lambda Tq^{N-m}} [1 - q^{N-m}] \quad (4)$$

The parameters are N, R, p, λ, C_F and C_v . T satisfying equation (4) is the optimal time T^* .

The expected cost T_c is given by:

$$\begin{aligned} T_c &= \sum_{k=1}^{\infty} [F^{(k)}(T) - F^{(k+1)}(T)] \left\{ C_v N (1 - q^k) + C_F \sum_{l=0}^{R-1} \binom{N}{l} (1 - q^k)^l (q^k)^{N-l} \right\} \\ T_c &= \sum_{k=1}^{\infty} e^{-\lambda T} \frac{(\lambda T)^k}{k!} \left\{ C_v N (1 - q^k) + C_F \sum_{l=0}^{R-1} \binom{N}{l} (1 - q^k)^l (q^k)^{N-l} \right\} \\ T_c &= C_v N e^{-\lambda T} \sum_{k=1}^{\infty} \frac{(\lambda T)^k}{k!} (1 - q^k) + C_F e^{-\lambda T} \sum_{k=1}^{\infty} \frac{(\lambda T)^k}{k!} \left\{ \sum_{l=0}^{R-1} \binom{N}{l} (1 - q^k)^l (q^k)^{N-l} \right\} \\ T_c &= C_v N - C_v N e^{-\lambda T p} + C_F e^{-\lambda T} \sum_{k=1}^{\infty} \frac{(\lambda T)^k}{k!} \left\{ \sum_{l=0}^{R-1} \binom{N}{l} (1 - q^k)^l (q^k)^{N-l} \right\} \\ T_c &= C_v N - C_v N e^{-\lambda T p} + C_F e^{-\lambda T} \sum_{k=1}^{\infty} \frac{(\lambda T q^{N-l})^k}{k!} \left\{ \sum_{l=0}^{R-1} \binom{N}{l} \sum_{m=0}^l \binom{l}{m} (-q^k)^{l-m} \right\} \\ T_c &= C_v N - C_v N e^{-\lambda T p} + C_F e^{-\lambda T} \left\{ \sum_{l=0}^{R-1} \binom{N}{l} \sum_{m=0}^l \binom{l}{m} (-1)^{l-m} [e^{\lambda T q^{N-m}} - 1] \right\} \end{aligned}$$

The optimal expected cost is therefore,

$$T_c^* = C_v N - C_v N e^{-\lambda T^* p} + C_F e^{-\lambda T^*} \left\{ \sum_{l=0}^{R-1} \binom{N}{l} \sum_{m=0}^l \binom{l}{m} (-1)^{l-m} e^{\lambda T^* q^{N-m}} \right\} \quad (5)$$

Numerical Illustration

The behaviour of optimal time and the expected cost are presented in Figure 1 and Figure 2, respectively. In the illustrations, N, λ, C_v, C_F and R are held constant and p is varied. The values of the parameters considered in the illustrations are: $N = 25, \lambda = 3, C_v = 200, C_F = 4750$ and $R = 10$. These parameter values are approximate values assumed for this illustration.

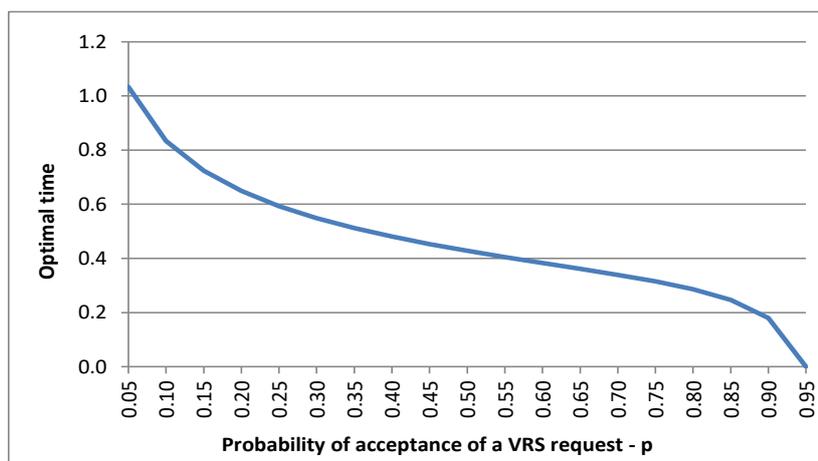


Figure1. Optimal time pattern for various p.

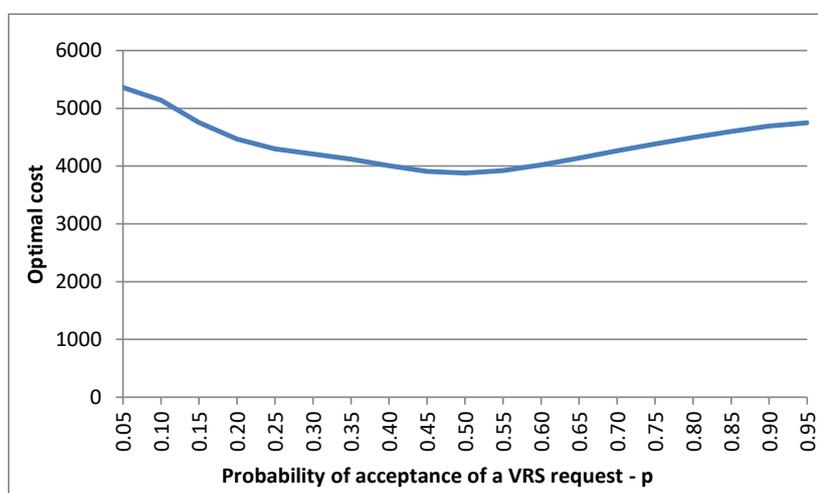


Figure2. Optimal cost pattern for various p.

From Figure 1, it is seen that the optimal time decreases with the increase in the probability of acceptance of the VRS request p . It is observed that for the set of given parameter values, the optimal time approaches zero at $p = 0.95$ as expected.

From Figure 2, it is seen that the optimal cost curve is concave in nature and hence a global minimum exists; it is obtained at $p = 0.5$. Also, the point $p=0.5$ in Figure 1 corresponds to a point of inflection.

Since a voluntary retirement scheme is intended to allow employees to retire early, the probability of acceptance of a voluntary retirement request p must be greater than 0.5. Further $p = 1$ is avoided to ensure that there is no mass voluntary turnover. The probability of acceptance of a VRS request p can also be decided by the organisation based on the budgeted cost.

Conclusion

An approach to determine the optimal time and optimal cost to withdraw a voluntary retirement scheme considering a probability of acceptance of a retirement request, cost due to announcing voluntary retirement scheme and cost to the organisation due to one-time special payments to those who voluntarily retire during the time period is discussed. A particular case where a Poisson process is assumed for the announcement of the voluntary retirement scheme and an illustration for the case is also presented.

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