Lyapunov-based Nonlinear Vibration Control of an Underactuated Aircraft Wing Model

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Abstract

This research presents a coupling nonlinear vibration control strategy for an underactuated wing model, characterized as a rigid link with translational and rotational springs that signify its structural stiffness. Stabilized by a single flap control surface, the system poses a complex control challenge. The control law design involves two steps: Step one employs coordinate transformation to create an equivalent simplified dynamic system with a new virtual control input through partial feedback linearization. Step two formulates a control law for this virtual input based on Lyapunov's theory, ensuring stability. Unlike conventional feedback linearization, this approach does not require assessing the system's internal stability. Moreover, the control law includes coupled terms related to the generalized coordinates of the target system with stabilized motion. MATLAB/SIMULINK simulations confirmed the effectiveness of the control structure in attenuating wing oscillations, despite the controller being non-adaptive and operating under assumed known wing model parameters.

Keywords: Nonlinear Coupling Control, Lyapunov Theory, Underactuated Wing, Partial Feedback Linearization.

1. Introduction

Much attention has been paid to flexible aircraft structures due to their improved efficiency. Lightweight aircraft lead to low drag and, hence, low thrust efforts from engines. However, this may reduce the aircraft's stability margins because of the interaction of aerodynamics, inertial forces, and elastic forces and moments. Besides, the presence of nonlinearities due to nonlinear geometry, elasticity of the coupled bending torsion, etc. could lead to different undesired scenarios such as flutter, limit cycle oscillations, and so on [1–3]. For the wing structure, different models are possible, such as plates or shells with complex cross sections or an equivalent system with a beam and flexible elements representing the structural stiffness and damping [4]. The work will be focused on the modeling of an aeroelastic wing based on an equivalent system.

In view of the above, a control structure is necessary to be integrated to attenuate the wing oscillations. In effect, two control categories are possible for vibration damping of the wing oscillations: passive and active control [5]. The passive control includes modifications in wing geometry, while the active control requires actuation systems (e.g., motors) and sensors to measure the oscillation signals. The second category is the aim of this study. Depending on the types of actuation systems (control surfaces), different situations are possible. Let us assume that the degrees-of-freedom DOFs of the wing are two, including pitch and plunge oscillations. If the number of control surfaces (e.g., flap control) is one, then an underactuation problem is encountered. If the number of control surfaces is two, then the system is fully actuated, and most conventional control approaches can be used. On the other hand, if strain-based actuation is added in line with flap control, then the system could be overactuated. This work is focused on the underactuated case, which can be considered the more challenging problem. The powerful tool to deal with this case is partial feedback linearization control; in the work of Strganac and his colleagues [6–8]. Partial feedback linearization is a technique that transforms only a subset of a system's states, allowing it to remain nonlinear while transforming the rest of the system into a linear one. This is useful when full linearization is not possible, such as in systems with strict input constraints or non-observability, where some states cannot be measured. By transforming the observable states, partial feedback linearization simplifies the design of a controller that can stabilize the system, ensuring a more stable system. Thus, the partial feedback linearization aims to partition the DOFs into two categories: active and passive DOFs. The control objective is to track and regulate the passive DOFs, while the internal dynamics (zero dynamics) associated with passive DOFs should be stable. In effect, the underactuated wing dynamics are similar to those of inverted pendulums [9], floating base robots [10], flexible base robots [11], and mobile robots [12]. It should be noted that the standard control law based on partial feedback linearization includes one set of active DOFs that should be controlled or regulated. Sure, exceptions are possible as made in works [11, 13].

In [11], a robust time-delay base decoupling approach is used to decouple the manipulator dynamics from the compliant base dynamics, and then a control law is designed that includes fast and slow response terms associated with manipulator joints and base generalized coordinates, respectively. In [13], a partial feedback linearization approach is used for decoupling the flexible link dynamics from the rigid link dynamics, and then a Lyapunov-based control law is used for designing the control law. The resulted control law includes generalized coordinates of flexible and rigid links together. This technique coincides with the energy-based control described in [14]. For more details on the control of compliant base robots, see, e.g., [15–17] and the references therein. In [18], function approximation technique-based adaptive feedback linearization is used for controlling an aircraft wing considering three different actuation modes: full, over, and under actuation.

This study explores a Lyapunov-based control law for damping vibrations in an underactuated wing. The wing is modeled as an equivalent dynamic system comprising a rigid link and two flexible elements that capture translational and torsional stiffness. A partial feedback linearization method is employed to decouple the active angular coordinate from the vertical passive coordinate. Subsequently, a nonlinear control law is developed based on Lyapunov stability to stabilize the motion of 2D wing model. The study is structured as follows: Section 2 discusses the dynamics and control law, while Sections 3 and 4 present simulation results and conclusions.

2. Methodology

The two primary degrees of freedom (DOF) wings experience during flutter are the bending mode (plunge) and the torsion mode (pitch). Flutter arises when the structure absorbs energy from airflow due to the interaction of elastic, inertial, and aerodynamic forces, resulting in various vibrational modes and instability. This phenomenon occurs when the damping ratio of the critical mode reaches zero at a specific airspeed. Figure 1 depicts a two-dimensional airfoil with two degrees of freedom (2-DOF): plunge and pitch. The wing can plunge downward and pitch upward around its elastic axis, while the control flap rotates downward around its hinge. This model forms the basis of quasi-steady aerodynamics. The following assumptions are assumed [6–8]:

1. The structural stiffness in vertical and torsional directions is linear.

2. The aerodynamic forces and moments are developed based on a quasi-static formula.
3. The system is underactuated with two degrees of freedom (DOFs) controlled by a single input—the flap angle—affecting the airfoil's aerodynamic properties and motion. Consequently, the presence of one flap and two DOFs renders the equation of motion underdetermined.

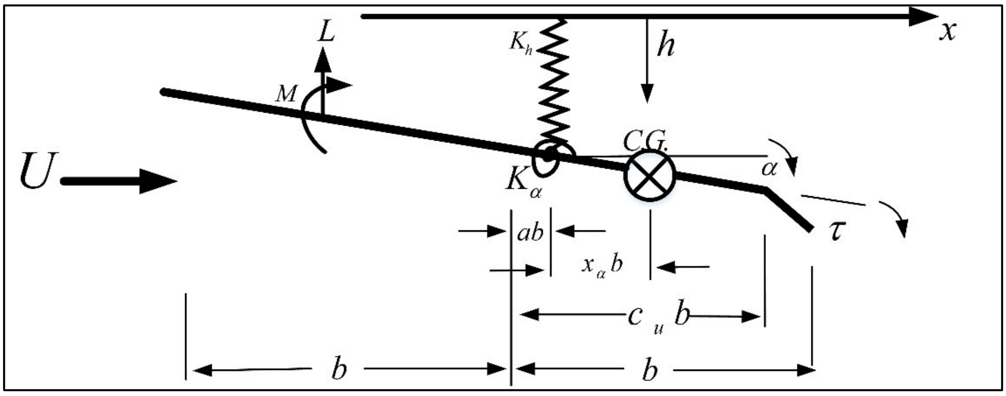

Figure 1. A Two-DOF Wing Model [18].

The equation of motion can be represented as follows [6-8]:

$$m_{11}\ddot{h}+m_{12}\ddot{\alpha}+c_{11}\dot{h}+c_{12}\dot{\alpha}+k_{11}h+k_{12}\alpha=-a_1\tau \quad \text{(1a)}$$

$$m_{21}\ddot{h}+m_{22}\ddot{\alpha}+c_{21}\dot{h}+c_{22}\dot{\alpha}+k_{22}\alpha=a_2\tau \quad \text{(1b)}$$

with

$m_{11}=m$, $m_{12}=mx_\alpha b$, $m_{21}=m_{12}$, $m_{22}=I_\alpha$, $c_{11}=c_h+\rho Ubc_l$, $c_{12}=\rho Ub^2c_l(\frac{1}{2}-a)$,

$c_{21}=\rho Ub^2c_m$, $c_{22}=c_\alpha-\rho Ub^3c_m(\frac{1}{2}-a)$, $k_{11}=k_h$, $k_{12}=\rho U^2bc_l$, $k_{22}=-\rho U^2b^2c_m+k_\alpha$

$a_1=-\rho U^2bc_l$, $a_2=\rho U^2b^2c_m$

where m is the mass of the wing system, x_α is a dimensionless parameter, b is a semi-chord reference length of the wing, I_α is the moment of inertia of the wing, c_h is a viscous damping coefficient related to the plunge coordinate, h, c_α is a viscous damping coefficient related to the pitch coordinate, α, k_h is a translational structural stiffness, k_α is a rotational structure stiffness, U is free stream velocity, c_l and c_m are steady aerodynamic coefficients, and τ is the control input representing the flap angle.

Partial feedback linearization is a control strategy that stabilizes underactuated mechanical systems by transforming them into linear systems. This approach adjusts the control input to influence specific system coordinates while leaving others unchanged, simplifying control design and analysis. When a system cannot be fully linearized, it can be separated into two interconnected subsystems: one linear and one nonlinear. The goal of partial feedback linearization in equations. (4a) and (4b) is to create a simpler equivalent system for further control design of the resulting virtual control input (u). In the analysis, the study utilizes Lyapunov's method to develop a suitable control law for the wing system. Using the concept of partial feedback linearization, let us define

$$\ddot{\alpha} = u \tag{2}$$

where u is a new control input that should be designed to ensure stability. Substituting Eq. (2) into Eq. (1a) to obtain the following control input

$$\tau = -\frac{1}{a_1}\left[m_{11}\ddot{h} + m_{12}u + c_{11}\dot{h} + c_{12}\dot{\alpha} + k_{11}h + k_{12}\alpha\right] \tag{3}$$

Equation (3) is highly coupled and will be nonlinear if the stiffness terms are inherently nonlinear. However, the proposed control works well for both linear and nonlinear problems.

With some mathematical manipulations, the following equivalent system is achieved:

$$\ddot{\alpha} = u \tag{4a}$$

$$\overline{m}_h\ddot{h} + \overline{c}_h\dot{h} + \overline{k}_h h + \overline{c}_\alpha\dot{\alpha} + \overline{k}_\alpha\alpha = -\gamma u \tag{4b}$$

where,

$$\overline{m}_h = m_{21} + \frac{a_2}{a_1}m_{11}, \quad \overline{c}_h = c_{21} + \frac{a_2}{a_1}c_{11}, \quad \overline{k}_h = \frac{a_2}{a_1}k_{11}, \quad \overline{c}_\alpha = c_{22} + \frac{a_2}{a_1}c_{12}, \quad \overline{k}_\alpha = k_{22} + \frac{a_2}{a_1}k_{12},$$

$$\gamma = \frac{a_2}{a_1}m_{12} + m_{22}$$

The next step is to design the virtual control input u based on Lyapunov theory, see the following theorem and lemma.

Lemma 1. Let $x^T = \begin{bmatrix}\dot{h} & \dot{\alpha}\end{bmatrix}$ and $P = \begin{bmatrix}\overline{m}_h & \gamma \\ \gamma & b_2\end{bmatrix}$, then $x^T P x = \overline{m}_h \dot{h}^2 + \gamma \dot{h}\dot{\alpha} + b_2 \dot{\alpha}^2$ is a quadratic form and positive definite if only if $\overline{m}_h > 0$ and $\overline{m}_h b_2 - \gamma^2 > 0$, where b_2 is a positive constant gain. The proof of the above lemma is easily made based on one of the tests of positive definite matrices.

Theorem 1. The dynamic modeling of the aircraft wing described in Eq. (1), with the closed loop dynamics presented in Eq. (4), and the following control law

$$u = \frac{1}{\gamma}\left[\left((\bar{c}_\alpha + \gamma \bar{c}_h) + \bar{k}_\alpha \alpha - \bar{k}_h h\right)\dot{h} + (\bar{c}_\alpha \gamma - d)\dot{\alpha} \quad + \left(\gamma \bar{k}_\alpha - b_1\right)\alpha\right] \tag{5}$$

where d is a positive constant parameter.

Proof.

According to the above lemma, let us consider the Lyapunov function along Eq. (4)

$$V = \frac{1}{2}\overline{m}_h \dot{h}^2 + \frac{\bar{k}_h}{2} h^2 + \frac{b_1}{2}\alpha^2 + \frac{b_2}{2}\dot{\alpha}^2 + \gamma \dot{h}\dot{\alpha} \tag{6}$$

where b_1 is a positive constant parameter. Taking the time derivative for the above equation leads to

$$\dot{V} = \overline{m}_h \dot{h}\ddot{h} + \bar{k}_h h\dot{h} + b_1 \alpha\dot{\alpha} + b_2 \dot{\alpha}\ddot{\alpha} + \gamma\left(\dot{h}\ddot{\alpha} + \dot{\alpha}\ddot{h}\right) \tag{7}$$

Substituting Eq. (4a) and Eq. (4b) into above equation to get

$$\dot{V} = \dot{h}\left[-\gamma u - \bar{c}_h \dot{h} - \bar{c}_\alpha \dot{\alpha} - \bar{k}_\alpha \alpha\right] + b_1 \alpha\dot{\alpha} + \bar{k}_h h\dot{h} + b_2 \dot{\alpha}(u) + \gamma \dot{h} u - \gamma \dot{\alpha}\left[\frac{\gamma}{\overline{m}_h} u + \bar{c}_h \dot{h} + \bar{c}_\alpha \dot{\alpha} + \bar{k}_\alpha \alpha\right] \tag{8}$$

With some manipulation for Eq. (6), we can get

$$\dot{V} = -\bar{c}_h \dot{h}^2 + \left[\left(b_2 - \frac{\gamma^2}{\bar{m}_h} \right) u - \bar{c}_\alpha \dot{h} - \gamma \bar{c}_h \dot{h} - \bar{c}_\alpha \gamma \dot{\alpha} \ - \gamma \bar{k}_\alpha \alpha + b_1 \alpha \right] \dot{\alpha} - \bar{k}_\alpha \alpha \dot{h} + \bar{k}_h h \dot{h} \tag{9}$$

According to theorem 1 and the associated control law proposed in Eq. (5), Eq. (9) becomes

$$\dot{V} = -\bar{c}_h \dot{h}^2 - d\dot{\alpha}^2 \leq 0 \tag{8}$$

LaSalle's invariance principle serves as a fundamental framework that demonstrates the uniform asymptotic stability of the closed-loop origin. This significant conclusion is evidenced through the meticulous mathematical expressions presented in equations (2) and (3), along with the essential feedback control law detailed in equation (5). As a direct result of this analysis, equation (8) emerges as semi-negative, thereby providing substantial confirmation of the controller's uniform asymptotic stability when viewed through the lens of Lyapunov theory, as elaborated in the referenced work [19]. Notably, this particular controller is ingeniously designed to consolidate both state variables into a unified control law. This innovative approach effectively removes the need to conduct a thorough verification of the stability of internal dynamics, a point that has been emphasized and supported in various previous studies conducted in this field. Thus, the implications of LaSalle's principle, in conjunction with the structure of the feedback control system, play a pivotal role in ensuring stability while simplifying the overall analysis required for control system performance. The proposed controller consists of two main steps. Step 1 uses feedback linearization to simplify the system's dynamics, making them more manageable. Step 2 develops a control law for the virtual control input, employing Lyapunov theory to ensure stability. This approach is more straightforward and user-friendly than alternatives like pure partial feedback linearization, energy-based control, flatness-based control, and feedforward-based methods, as referenced in [20-22]. Overall, the approach aims to streamline control processes and improve system behavior.

3. Results and Discussion

In this section, the validity of the proposed control structure of the previous section is proved by using a simulation example on a 2D wing model described previously in Figure. 1. The physical parameters used in the simulation experiment are described as follows.

$a = -0.8, b = 0.14\ m, x_\alpha = 0.2, c_h = 28\ N\frac{m}{s},\ c_\alpha = 0.04\ Nm,\ c_l = 6.3,$

$c_m = (0.5 + a)c_l, c_{l_u} = 3.4,\ k_h = 2845\ N/m, k_\alpha = 3\ N.m/rad, I_\alpha = 0.06\ kg.m^2,$

$m = 2\ kg, \rho = 1.3\ kg/m^3.$

A constant control gain is implemented, specifically setting $b_1 = 1$, $b_2 = 2$, and $d = 1$. These values are chosen because they satisfy the necessary conditions stipulated in Lemma 1. In order to induce vibrations within the wings, the plunge and pitch coordinates are initialized at specific values of (0.01 m for plunge and 0.1 rad for pitch). To analyze the system's behavior under these conditions, we conducted a simulation experiment using MATLAB/SIMULINK. This simulation effectively utilized the controller structure as characterized in Theorem 1, allowing for a detailed examination of the system dynamics. The results derived from this simulation indicate a noteworthy effectiveness in the attenuation of wing oscillations, which is visually presented in Figures 2 through 4.

In Figures 2 and 3, a comparative analysis of the open-loop and closed-loop oscillations specifically for pitch and plunge coordinates is provided. The illustrations clearly demonstrate that the proposed controller exhibits significant capability for dampening the oscillations, thereby leading to a substantial reduction in the response error. This comparative analysis highlights the enhancements in performance brought about by the utilization of the proposed control strategy. Further, Figure 4 presents a visual representation of the control input signals, noting the saturation effects that occur within the system. These input signals are constrained within the range of 0.5 to -0.5 rad, which is pertinent for ensuring that feasible flap angles are maintained throughout the operation.

It is also crucial to emphasize that the controller implemented are non-adaptive in nature. This means it operates under the assumption that the parameters of the wing model are known and constant. In real-world applications, however, it is essential to employ a system identification method, which is a process that would allow for the accurate estimation of these physical parameters. This process is vital for improving control strategies and achieving optimal performance in practical scenarios where model parameters may vary or be uncertain.

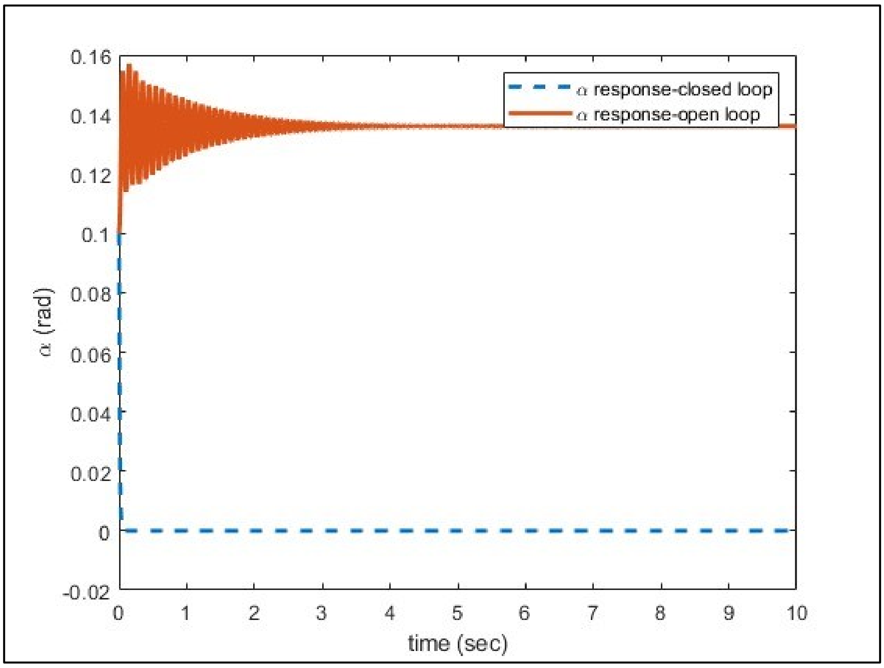

Figure 2. Pitch Response

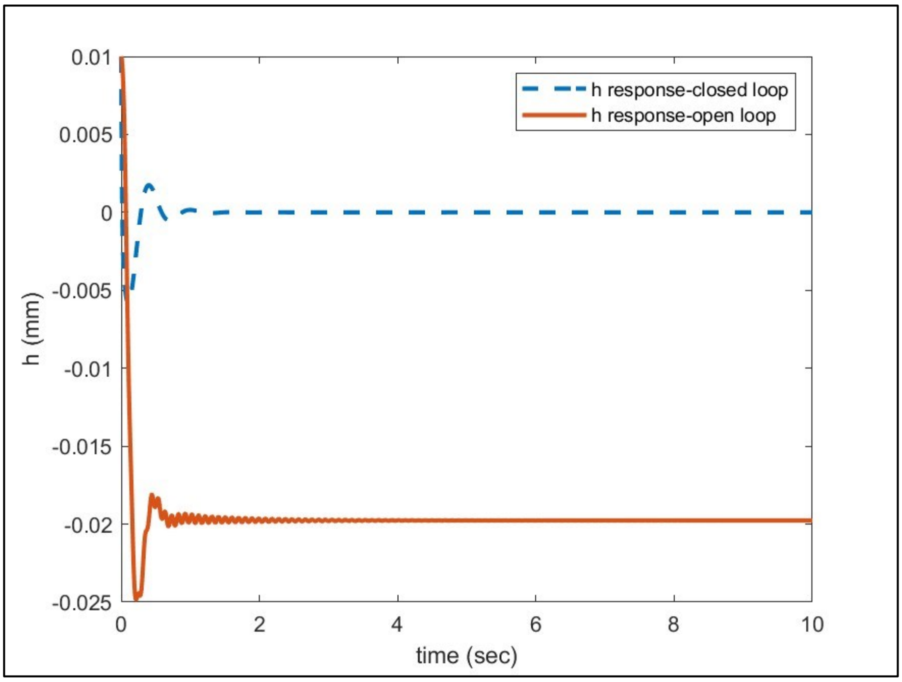

Figure 3. Plunge Response

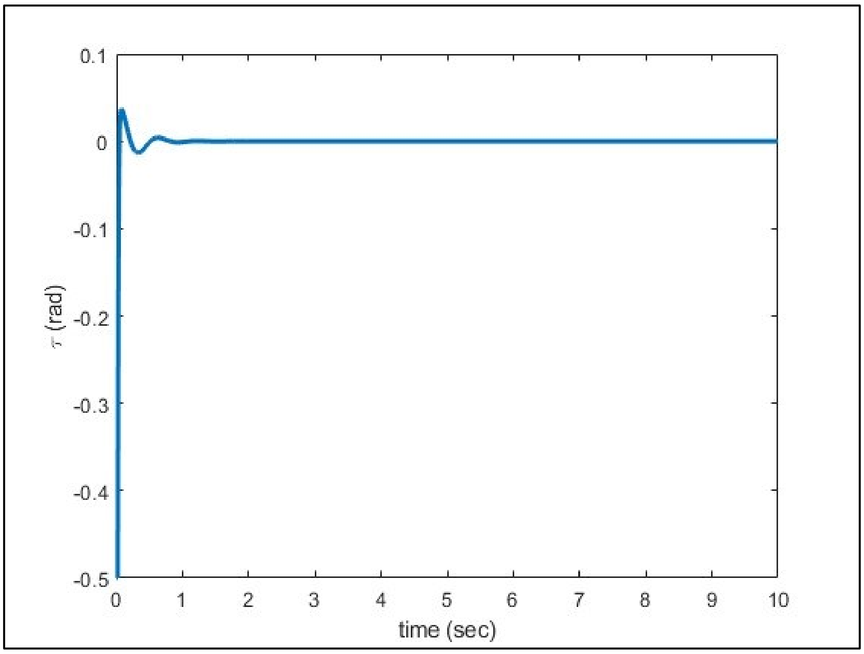

Figure 4. Control Input with Anti-Windup Control.

4. Conclusions

This study introduces and discusses a nonlinear control strategy specifically designed for an underactuated wing model. The control law currently in use exhibits significant nonlinearity, which allows it to effectively avoid the requirement for maintaining internal stability in specific coordinates of the wing system that are typically necessary when employing partial feedback linearization techniques. Despite its innovative approach, the existing controller is limited in its capabilities, as it lacks the necessary adaptability to address and manage system vibrations that arise due to unknown physical parameters present in the model. This shortcoming highlights an important area for improvement. Therefore, it is essential for future research to concentrate on the development of an adaptive control version of this strategy. Such advancements would enhance the algorithm's practicality and effectiveness in real-world applications, addressing the challenges posed by variable parameters and dynamic conditions within the system.

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Author's biography

Dr. Hayder Al-Shuka, born in 1979 in Baghdad, Iraq, earned his PhD in 2014 from RWTH Aachen University, Germany, specializing in Mechanism Theory, Machine Dynamics, and Robotics. He was a lecturer in the Mechanical Engineering Department at Baghdad University from 2006 to 2017, later completing a postdoctoral fellowship at Shandong University in China from 2017 to 2019. Since 2020, he has served as an assistant professor in the Department of Aeronautical Engineering at Baghdad University. His research focuses on biped robot walking patterns, control, and adaptive control for robotic manipulators, exoskeletons, and prosthetics.

Burkhard J. Corves was born in Kiel, Germany, in 1960. He earned his diploma and PhD in mechanical engineering from RWTH Aachen University in 1984 and 1989, respectively. From 1991 to 2000, he worked at a Swiss engineering company while also holding teaching assignments at RWTH Aachen University. Since 2000, he has served as a university professor and director of the Department of Mechanism and Machine Dynamics at RWTH Aachen University. His research interests include the kinematics and dynamics of mechanisms and robots.